ESSAYS IN MACROECONOMICS AND PUBLIC FINANCE

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Essays in Macroeconomics and Public Finance

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Abstract

The dissertation examines how fiscal policies adjust to economic states in a growth model where productions are mobile across jurisdictions and the corresponding consequences. In my work, I study the properties of optimal state-level corporate and labor income tax rates and how shocks in the federal tax rates affect the economy; and I endogenize the federal-level fiscal policies in a Stackelberg game setting where the federal government is the leader and the states are the followers.

In “Fiscal Competition and Federal Shocks”, I answer such the question of “how a shock to federal tax rate affect the macro-economy”. The innovation is that I take into account the effects of factor mobility, state-federal interaction, and state-state interaction on the transmission mechanism of the federal shocks. By using the U.S. data set, I find the evidence that state-level tax rates will respond to changes in federal tax rates (known as vertical competition) and the neighboring state’s policies (known as horizontal competition). To rationalize this finding, I develop a two-region growth model with benevolent state governments, integrated capital market, and sticky migration. My quantitative result indicates that omitting the endogenous responses of state-level policies leads to

significant difference in response to a federal shock. This means that the central policy make has to consider the intergovernmental fiscal relations when designing federal fiscal policies.

In “Optimal Policies in a Federation”, I examine the optimal federal and state fiscal policies in a dynamic macro model with policy commitment, integrated capital market, and inter-state migration. In modeled governance system, the federal government is the Stackelberg leader, the state governments are the followers and take the leader’s policies as given. In the interior-point steady-state, the overall tax rate on corporate income is zero. However, the leader and followers impose different tax rates. The leader levies a positive and high tax rate, the followers levy negative tax rates. The zero (overall) tax rate result holds when the states are heterogeneous in their TFPs. If the federal government has to impose the same labor income tax rate on the states, the federal tax rates are independent of the degree of inequality and each state has a zero overall corporate tax rate. IF the federal labor tax system is nonlinear, the states impose different tax rates. But the tax-base-weighted overall tax rate in the economy is still zero. In addition, I find that increasing the federal corporate tax rate is the optimal response to foreign country’s TFP becomes higher.

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I tried and I did it.

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Chapter 1

Fiscal Competition and Federal Shocks

1.1 Introduction

Taxation is an important component of fiscal policies. Changes in taxes not only affect government finance but also influence economic activities. To evaluate the economic effect of a tax shock, existing researches build either neoclassical models[1] or New Keynesian models[2] in which there is a unique fiscal authority. However, such a setting may be unrealistic in a federation where actual fiscal authorities include the federal government, state, and local governments, as it fails to take into account the interaction between different governments. This paper studies the effect of a federal tax rate shock on the economy under the condition that state fiscal policies will respond to the shock. On this basis, I

[1] e.g. Yang (2005), Leeper, Walker, and Yang (2010), Mertens and Ravn (2010), Chahrour, Schmitt-Grohé, and Uribe (2012).

[2] e.g. Traum and Yang (2011, 2015), Leeper, Richter, and Walker (2012), Chen, Leeper, and Leith (2015), Leeper and Traum (2017), Eusepi and Preston (2018).

analyze the economic influence of these responses.

This paper highlights two channels that shape the states' responses to a federal tax change. First, since both federal and state governments impose taxes on the economy, state tax rates will react to changes in federal tax rates because these changes affect a state's tax base — a channel known as *vertical fiscal competition*. Second, since production factors (capital, labor) are mobile across states, competing for these factors makes state tax rates interdependent — a channel known as *horizontal fiscal competition*. Horizontal competition has the potential of altering the magnitudes of vertical competition.[3] In the end, the private sector's decisions will react to the net effect of the federal and state tax changes. Particularly, I use corporate and personal income tax to illustrate these two channels in this paper.[4]

This paper answers three questions to shed light on the two channels above. How do the state corporate and personal income tax rates respond to a federal tax change? To what extent do these responses matter for the effect of the federal tax change? Which role does horizontal competition play in these responses?

To answer the first question, I start by providing empirical evidence on vertical fiscal competition, which only has been studied by a few papers.[5] Using

[3] Suppose in response to a federal corporate income tax rate change, New Hampshire decides to cut the state corporate income tax rate. This provides Massachusetts an incentive to cut its own corporate income tax rate to prevent the business from out-flowing. On top of that, Massachusetts also responds to the federal tax change.

[4] Because these two taxes are important components of both federal and state revenue. They are also instruments for inter-state competition because they affect the private sector's behaviors directly.

[5] Hayashi and Boadway (2001) uses provincial data in Canada and shows that business tax rates at the provincial level respond negatively to the federal business tax rate. Reingewertz (2018) finds that state corporate tax rates are decreasing in federal corporate tax shocks, but increasing in federal non-corporate tax shocks.

quarterly data from 1950Q1 to 2006Q4, I estimate the local projection (LP) impulse responses of state corporate and personal income tax rates to a shock in federal corporate and personal income tax rates. The result shows that in response to a 1% increase in the federal corporate income tax rate, after a year, the average state corporate income tax rate falls by 0.15%, and the average state personal income tax rate raises by 0.02%. When the shock is in the federal personal income tax rate, after a year, the average state corporate income tax rate raises by 0.5%, and the average state personal income tax rate falls by 0.02%. Regarding horizontal fiscal competition, there has been plenty of empirical evidence.[6] Using an updated state-level data set, my work finds an inter-state correlation of 0.45 for corporate income tax rates and the number for personal income tax rates is -0.32, which are consistent with existing results.

To find the reason behind the empirical findings and provide a foundation for answering the rest of the questions — to what extent do states' responses matter for the effect of a federal tax change; which role does horizontal competition play in these responses — I develop a dynamic two-state model on vertical and horizontal fiscal competition and study the optimal state policies.

In the modeled economy, households are labor and asset owners who make migration, labor supply, and consumption decisions. A household moving from

[6]For example, Rork (2003) provides comprehensive results on different taxes. He finds that state taxes with mobile tax bases (e.g., corporate income tax) have positive inter-state correlations; however, state taxes with immobile tax bases (e.g., personal income tax) have negative inter-state correlations. The positive intergovernmental correlation of corporate income tax rates have been documented by works such as Heyndels and Vuchelen (1998), Buettner (2001), Revelli (2001), Hayashi and Boadway (2001), Devereux, Lockwood, and Redoano (2008), Charlot and Paty (2010), and Chirinko and Wilson (2017) (it shows that the regression coefficient for the neighbor's current period tax rate is significantly positive) in different countries or regions, Devereux and Loretz (2013) provides a review. Meanwhile, the negative inter-state correlation of personal income tax rates is also found by Parchet (2019), which studies the local personal income taxed in Switzerland.

one state to another will receive a shock drawn from a random distribution, which implies labor is costly mobile. A nationwide investor allocates capital between firms in the two states costlessly (hence capital is freely mobile), makes saving decision,[7] and pays dividends to households. Since the focus is on state fiscal policies, I assume federal corporate and personal income tax rates are exogenous and governed by AR(1) processes. In each state, the benevolent state government collects corporate and personal income taxes and issues bonds to finance public goods and bond repayment. After households have settled down, the two governments set fiscal policies strategically in a Cournot game in every period to maximize the average welfare of current residents.[8]

By solving the model, this paper shows that the optimal state tax rates depend on federal tax rates. A higher federal corporate income tax rate increases the marginal effect of capital on a state's budget constraint but decreases the marginal effect of labor. This is because a higher amount of capital (labor) lowers (raises) the marginal product of capital (MPK), hence decreases (increases) the federal corporate tax: federal corporate tax rate$\times$(MPK$-$depreciation rate)$\times$ capital. As a result, the state planner has an incentive to lower the state corporate income tax rate and raise the state's personal income tax rate.

Instead, a higher federal personal income tax rate decreases the marginal effect of capital on a state's budget constraint but increases the marginal effect of labor. This is because a higher amount of capital (labor) raises (lowers) the marginal product of labor (MPL), hence increases (decreases) the federal per-

[7]Letting the investor make the saving decision avoids household heterogeneity and simplifies the problem.

[8]This paper looks at the perfect Markov equilibrium. It means that there is no policy commitment and the equilibrium policies are time-consistent. As mentioned by Kehoe (1989), horizontal competition can be viewed as a partial commitment.

sonal tax: federal personal tax rate × MPL × labor. Therefore, a state planner will increase the state corporate income tax rate and decrease the state's personal income tax rate.

Apart from federal tax rates, this paper finds that a state's corporate income tax rate is increasing in the neighboring state's tax rates. An increase in the neighboring state's corporate or personal income tax rate causes capital to flow into the home state[9] and generates two effects. First, the capital inflow increases the total cost of capital in the home state, which motivates the home state to increase the corporate income tax rate as it lowers the interest rate. Second, having more capital in the home state, the tax base of corporate income tax is less sensitive to the tax rate change, which provides another incentive to raise the corporate income tax rate.

Additionally, this paper finds that a state's personal income tax rate is decreasing in the neighboring state's tax rates. As described above, an increase in the neighboring state's corporate or personal income tax rate causes capital to flow into the home state and relaxes the home state's budget constraint. In the home state, the relaxed budget constraint decreases the shadow price of pubic goods and increases the marginal rate of substitution (MRS) of household consumption to public good. That is, the home state's planner has the incentive to sacrifice more public goods to gain one more unit of average household income, which means the home state's personal income tax rate should decrease.

To illustrate how horizontal competition shapes a state's responses of tax rates to a federal shock in the model, here I use the state corporate income tax

[9] Recall that increasing the personal income tax rate of the neighboring state reduces the neighboring state's labor supply, which also reallocates capital towards the home state.

rate as an example. Intuitively, a federal shock brings three effects: i) *direct effect* — keep the neighboring state's tax rates fixed, the home state's corporate tax rate will react directly to the shock itself; ii) *amplification effect* — as the neighboring state also responds to the shock, the home state's corporate tax rate will react to the resulting change in the neighboring state's corporate tax rate, which amplifies the direct effect as the inter-state correlation is positive;[10] iii) *cross-tax effect* — the home state's corporate income tax rate will also react to the resulting change in the neighboring state's personal tax rate. Since the state personal and corporate tax rates respond to the same federal shock in the opposite directions, the cross-tax effect tends to moderates the direct effect. Therefore, the influence of horizontal competition is ambiguous.

In the quantitative analysis, I calibrate the model parameters[11] to match moments in the U.S. including the size of state personal income tax relative to corporate income tax, federal tax revenue as a fraction of GDP, state public expenditure as a fraction of GDP, state government bond to GDP ratio and its standard deviation, as well as annual inter-state out-migration rate. With the model in hand, I study the economic effects of federal tax rate shocks.

For each federal tax rate shock, this paper computes the impulse responses in three different models. The invariant-state-policy (ISP) model, which fixes the state fiscal policies at the baseline model's steady state levels and excludes both vertical and horizontal fiscal competitions. The immobile-factor (IF) model, which endogenizes the state fiscal policies but assumes that factors are immo-

[10]In the dimension of the state personal income tax rate, the second effect becomes the *dampening effect* because the inter-state correlation is negative.

[11]The key calibrated parameter include the steady state federal corporate and personal income tax rate, preference for public goods, the cost of issuing state government bond, and the mean and deviation of migration shock.

bile, hence only considers vertical competition. The baseline model, which takes both types of fiscal competitions into account.[12] Comparing the impulse response of output in the three models provides valuable information. The difference between the ISP model and the IF model shows the effect of *direct vertical competition* — vertical competition without the influence of factor mobility; the difference between the IF model and the baseline model tells the role of horizontal competition; the difference between the ISP model and the baseline model displays the net influence of state fiscal policies' responses. Specifically, this paper puts attention on both the on-the-impact effect and the transitional path.

Consider a 1% cut in the federal corporate income tax rate. In the ISP model output changes by 0% on the impact since capital is predetermined and labor does not change due to the invariant state tax rates. In the coming periods, output gradually increases because the lower tax rate on corporate income encourages capital investment and labor supply (since the capital-labor ratio rises). Therefore, there exists a hump shape in the impulse response.

In the IF model, in response to the same federal shock output increases by 0.065% on the impact. This is because the state personal income tax rate decreases by 0.2%, which encourages labor supply. Instead of the hump shape, output in the IF model falls monotonically in the following periods. The key reason is the over-response of state corporate income tax rate, whose value increases by 1.37% on the impact and gradually moves back. This over-response raises the overall tax rate on corporate income above the pre-shock level, hence reduces capital investment and output.[13]

[12] The rest of the settings are the same in all three models.

[13] Since capital depreciation is deductible, one can show that the tax base for corporate income tax is (MPK − depreciation rate) instead of MPK for each unit of capital. As a result, a marginal

In the baseline model, output increases by 0.08% on the impact, which is bigger than that in the IF model. Because the cross-tax effect dominates the dampening effect, the response of state personal income tax rate decreases from -0.20% in the IF model to -0.24%, which gives a bigger labor supply increase. In the following periods, however, output falls faster than the IF model does. This difference attributes to the state corporate tax rate. Since the amplification effect dominates the cross-tax effect, the response of state corporate tax rate is higher than the corresponding value in the IF model. On the impact, the increase is 1.77% in the baseline model while the number in the IF model is 1.34%. This additional raise leads to an extra decrease in capital investment, which makes output drop faster.

In sum, the responses of state fiscal policies decrease the cumulative output effect of a federal corporate tax cut from 1.05% in the ISP model to 0.11% in the baseline model.[14] Inside of this change, 4/5 attributes to direct vertical competition, and 1/5 attributes to horizontal competition.

Consider a 1% cut in the federal personal income tax rate. In the ISP model which shuts down the responses of state fiscal policies, output increase by 0.33% on the impact because the lower tax rate on personal income encourages labor supply. After that, output falls towards the steady state as the shock fades.

In the IF model which only includes direct vertical competition, the same federal shock results in an increase of 0.10% in output on the impact. Relative to the ISP model, the force that moderates the initial output response is an increase

change in capital only induces a small change in tax revenue. Therefore, the state planner has an incentive to enlarge the response of the corporate tax rate.

[14] Let $\hat{y}_t$ be the percent deviation from steady state in period t, the cumulative effect of a shock $= \sum_{t=0}^{\infty} \beta^t \hat{y}_t$, where β is the discount factor.

of 0.70% in the state personal income tax rate. Such an increase partially offsets the federal personal tax rate cut, hence gives a smaller increase in labor supply and output. In the following periods, the response of output exhibits a hump shape. This is because the state corporate tax rate decreases in response to the shock (on the impact it drops by 3.20%), which encourages capital investment and keeps output increasing.

In the baseline model, output increases by 0.13% on the impact, which is higher relative to the IF model. This difference comes from the negative interstate correlation of state personal tax rate, which dampens its response from 0.70% in the IF model to 0.63% and encourages more labor supply. Similar to the IF model, the response of output also exhibits a hump shape in the baseline model. However, the peak comes earlier because the increase in capital investment is smaller. Since the cross-tax effect dominates the amplification effect, the state corporate tax rate decreases by less relative to the IF model. On the impact, the reduction is 2.14%, and the corresponding number in the IF model is 3.20%.

In sum, the responses of state fiscal policies increase the cumulative output effect of a federal personal tax cut from 1.75% in the ISP model to 2.40% in the baseline model. Introducing direct vertical competition increases the cumulative effect by 0.79%. On this basis, adding horizontal competition decreases the effect by 0.13%, around 1/5 of the total change.

As a counterfactual exercise, I study the economic effect of the 2017 federal tax reform. I construct the reform as a permanent cut in the federal corporate income tax rate combined with a temporary cut in the federal personal income tax rate. The sizes are 3.34% (about 1/3 of the steady state tax rate) and 2.24% (about 1/6 of the steady state tax rate) respectively. I find that the tax reform stimulates the short run output by 0.64%, which is attributed to the lower overall tax rate on personal income. However, the reform decreases the long run output by 0.31%. The reason is that the over-response of state corporate income tax rate decreases capital investment.

Related Literature. The current paper relates to several strands of literature. First, it relates to a large body of literature that analyzes the economic effects of tax shocks in general equilibrium models. Researches in this field either build neoclassical models (e.g. Yang, 2005; Leeper, Walker, and Yang, 2010; Mertens and Ravn, 2010; Chahrour, Schmitt-Grohe, and Uribe, 2012) or New Keynesian models (e.g. Traum and Yang, 2011; Leeper, Richter, and Walker, 2012; Traum and Yang, 2015; Chen, Leeper, and Leith, 2015; Leeper and Traum, 2017; Eusepi and Preston, 2018) in which there is a unique fiscal authority to implement their studies. This paper contributes to this group of literature by introducing two layers of government (federal government and state governments) and taking into account the impacts of states' responses on the economy. Under the single-fiscal-authority assumption mentioned above, the private sector only reacts to the tax shock itself. In contrast, this paper provides a more comprehensive transmission mechanism of a federal tax shock: i.e. the federal shock will result in changes in the state corporate and personal income taxes, and the private sector will react to the net effect of the federal and state tax changes.

Second, this paper also relates to the literature that studies the impact of vertical fiscal competition. Keen (1998) and Dahlby and Wilson (2003) construct a static model and discuss the externality of changes in the federal taxes. Relative to these two papers, my framework builds vertical competition in a dynamic framework, which allows one to explore the effect of a federal tax change on state-level public finance over time. Keen and Kotsogiannis (2002, 2004) establish a two-period model and study the efficiency (welfare) consequences of federalism. Specifically, both horizontal and vertical competitions are modeled in these two papers, which is similar to the present paper. Instead of efficiency, I focus on the economic implication of fiscal competitions, i.e. facing a federal tax shock, how states' responses of fiscal policies, which is shaped by horizontal and vertical competitions, affect the response of output. Moreover, I include both corporate and personal income taxes in the analysis, while the above two papers only consider taxes on capital.

Third, this paper builds upon horizontal fiscal competition models pioneered by Zodrow and Mieszkowski (1986), Wilson (1986), and Wildasin (1988), Bucovetsky and Wilson (1991), and Bucovetsky (1991).[15] They conduct studies in a static environment. Literature considering dynamic horizontal competition similar to mine includes: Mendoza and Tesar (2005), Quadrini (2005), Klein, Quadrini, and Rios-Rull (2005), Gross (2014), and Gross, Klein, and Makris (2017). These papers study the optimal tax rates under the condition of capital market integration using different equilibrium concepts. Mendoza and Tesar (2005) models horizontal competition as a one-shot game such that tax rates are time-invariant. Fiscal policies are time-varying and policy commitment is available in Gross (2014) and Gross, Klein, and Makris (2017). They show that in the steady state, the optimal tax rate on capital should be zero, which is

[15] Keen and Konrad (2013) provides a thorough review.

consistent with Chamley (1986) and Judd (1985). The present paper is close to Quadrini (2005) and Klein, Quadrini, and Rios-Rull (2005), who analyze the Markov perfect equilibrium, i.e. fiscal policies are time-consistent but there is no policy commitment. Relative to these two papers in which the optimal combination of tax rates is determined by grid search, I provide the formulas for optimal state tax rates and discuss the intuition behind them. Besides, both capital and labor are mobile across jurisdictions in my framework.

Last but not the least, to model the competition for households (labor), interstate migration is another element of this paper. A large body of literature in this field studies the determinants of worker distribution and the corresponding economic implications (e.g. Rosen, 1979; Roback, 1982; Moretti, 2004; Albouy, 2009; Coen-Pirani, 2010; Farhi and Werning, 2014; Hsieh and Moretti, 2019). This group of papers assumes local fiscal policies are exogenous in affecting worker inflows. Relative to the above papers, this paper endogenizes states' fiscal policies and shows that migration affects the borrowing incentive of the state planners. As a result, it matters for the setting of state tax rates and the provision of local public good. Armenter and Ortega (2010) and Deng (2019) also assume the states can choose their policies. They treat states as small islands, hence a state's policies only depend upon its state variables. Differently, I treat states as "big players" in this paper, which means that a state's policies also depend on other states' choices. Such a setting allows one to study the impacts of policy interactions.

Layout. The rest of this paper is organized as follows: section 2 provides the empirical on vertical and horizontal fiscal competitions; section 3 develops the dynamic fiscal competition model and discusses the optimal state policies; section 4 quantifies the model and conducts quantitative exercises; section 5 concludes.

1.2 Empirical Findings

In this section, I present two groups of empirical evidence. The first group relates to the responses of state corporate and personal tax rate to a federal tax rate shock (vertical competition). The second group is the inter-state correlation of state corporate and personal tax rate (horizontal competition).

1.2.1 Evidence on Vertical Competition

In this subsection, I explore how state corporate and personal tax rates respond to changes in the federal tax rates using the U.S. time series data. To avoid possible miss-specification, I use local projection (LP) proposed by Jordà (2005).

$$\tau_{t+h} = \mu_h + \beta_h o_t + \sum_{s=1}^{S} \delta'_{h,s} y_{t-s} + \gamma'_h x_t + \xi_{h,t}$$

$$h = 0, 1, 2, ..., H$$

In the above equation, τ is the cross-state average tax rate;[16] o_t is the federal tax rate at period t and θ_h represents the responsiveness of state tax rate at horizon h; vector y includes state and federal tax rates, real GDP per capita, tax base of corporate and personal tax, state government expenditure per capita, state liabilities per capita; vector x_t includes linear and quadratic time trends.

[16] In this subsection, the denominator of corporate is profit.

Besides, I use narrative shocks in Mertens and Ravn (2013) and federal debt per capita as instrumental variables. The time window is 1950Q1-2006Q4. Figure 1.1 displays the LP impulse responses of federal tax rate shocks.

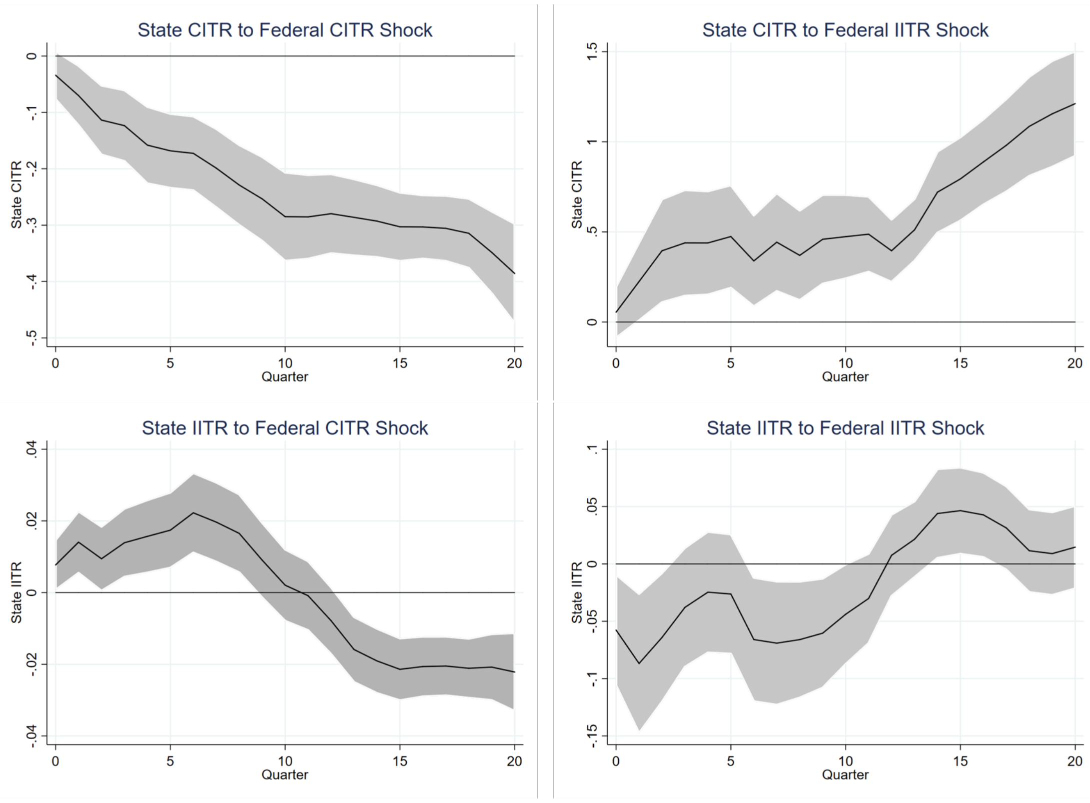

Figure 1.1: Effect of Federal Tax Rates on State Corporate and Personal Tax Rates

Looking at figure 1.1, shocks in federal tax rates have different effects on state-level tax rates. In response to a positive shock in federal corporate tax rate (the first column), a state will decrease corporate tax rate but increase personal tax rate. After a year, the responses are -0.15% and 0.02% respectively. If the positive shock is in federal personal tax rate (the second column), a state will increase corporate tax rate but decrease personal tax rate. After a year, the responses are 0.5% and -0.02% respectively. Existing literature usually focuses on the vertical competition on corporate income tax, my findings are consistent with Hayashi and Boadway (2001) and Reingewertz (2018).

1.2.2 Evidence on Horizontal Competition

Figure 1.2 shows the standard deviations of state corporate (the black-solid line) and personal (the red-dotted line) tax rates.[17] Starting from the 1970s, the dispersion of state corporate tax rates declines with some bump,[18] while the dispersion of state personal tax rates remains almost the same. This figure provides a raw evidence that state corporate and personal tax rates have different spatial relationships.

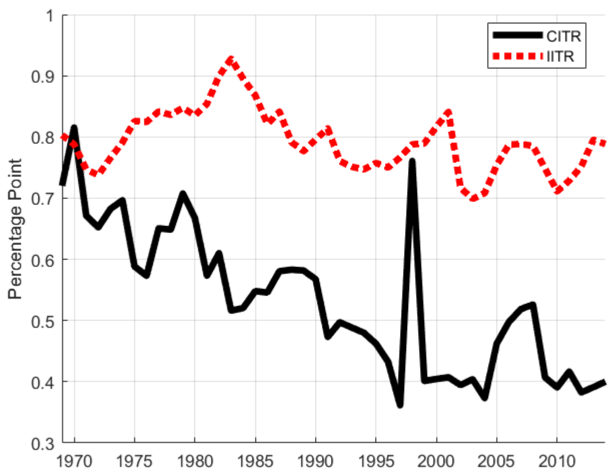

Figure 1.2: The Standard Deviation of State Corp. and Pers. Tax Rate

To explore the interactions of tax rates among states, I estimate the following empirical specification (with $z = \text{Corporate}, \text{Personal}$):

$$\tau^{z}_{i,t} = \alpha + \beta\tau^{z}_{-i,t} + x'_{i,t}\gamma + \lambda_t + \delta_i + \varepsilon_{i,t},$$

in which $\tau^{z}_{-i,t}$ is the competitor's corresponding rate; $x_{i,t}$ is the vector of control

[17] They are the average tax rates, computed by dividing tax revenue by the tax base. For corporate tax rates, the tax base is the gross operating surplus of private industries because the lack of state-level profit data. At the aggregate level, gross operating surplus is approximately three times as much as profit. The data contains states excluding AK, FL, HI, NV, SD, TX, WA, WY.

[18] For state corporate tax rate, the huge lift in 1998 is caused by UT, VT, WV increasing their rates largely.

variables; λ_t denotes the time effect; δ_i denotes the state effect. β is the coefficient of interest: a positive β suggests tax rates are strategic complements; a negative β suggests tax rates are strategic substitutes.

Table 1.1 reports the result (see appendix for the details). First, if the competitor increases its corporate tax rate by 1 percentage point, a state will increase corporate tax rate by 66 basis points. Second, if the competitor increases its personal tax rate by 1 percentage point, a state will decrease personal tax rate by 1.6 points. These findings are consistent with literature such as Rork (2003) and Parchet (2019).

Table 1.1: Empirical Evidence on Horizontal Competition

Variables	Corporate Tax Rate	Personal Tax Rate
$\tau_{-i,t}$	0.4500	-0.3187
	$(0.2159)^{**}$	$(0.1434)^{**}$
IV	Yes	Yes
Control	Yes	Yes
Fixed Effect	Yes	Yes
Time Effect	Yes	Yes
$N \times T$	924	924

Standard errors in parenthesis; $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$; The sample covers 42 states who have both CIT and IIT from 1992 to 2014.

1.2.3 Remarks

The exercises in this section lead to the following findings. First, the state corporate and personal tax rates respond differently to the same federal tax rate shock. To be specific, an increase in the federal corporate tax rate will lead to a decline in the state corporate tax rate and a rise in the state personal tax rate. In contrast, an increase in the federal personal tax rate will lead to a rise in the state corporate tax rate and a decline in the state personal tax rate. Second, the inter-state correlation of state corporate tax rates is positive, and the corresponding value for state personal tax rates is negative. These findings emphasize the need for writing a competition model that discriminates the role and property of state corporate and personal taxes.

1.3 The Model

This section studies the dynamic fiscal competition problem in a two-state economy. The main elements include: i) a federal government who imposes taxes on corporate and personal income; ii) two state governments who tax corporate and personal income in their jurisdictions and issue bonds to finance public goods;[19] iii) a continuum of households with a total population of $\bar{n}$;[20] iv) two perfectly competitive firms (one in each state); v) one nationwide investor who is in charge of capital allocation, investment/saving, and pays dividends to households.

Both households (labor) and capital are mobile in this modeled economy. A household moving from one state to another will receive a migration shock drawn form some random distribution, therefore labor is costly mobile. The investor can allocate capital between firms in the two states costlessly, there fore capital is freely mobile.

In the following parts, I am going to introduce the timing of the model, exhibit each element, and discuss the optimal fiscal policies.

[19]The ideal setting would be 50 states. Relative to its benefit, the cost — explosion of state variables — is too high.

[20]$\bar{n}$ is set to 2 in the quantitative exercise.

1.3.1 Timing

Since the focus is on state fiscal policies, I assume federal corporate and personal income tax rates are exogenous and governed by AR(1) processes. In each period, the timing is as follows:

1. The federal and regional shocks realize, households learn their assigned migration shocks;
2. Households make migration decisions;[21]
3. The state governments set fiscal policies;
4. The investor allocates capital, households work, and firms produce;
5. The investor makes investment/saving decisions and pays dividends, and households consume.

Household

Let $\mathbf{X}_t$ be the vector of state variables. Upon observing $\mathbf{X}_t$, the migration problem of a household initially living in state i and receiving a migration shock κ reads as:

$$\max_{m=0,1} (1-m)V_i\left(\mathbf{X}_t\right) + m\left[V_{-i}(\mathbf{X}_t) - \kappa\right].$$

$V_i(\mathbf{X}_t)$ and $V_{-i}(\mathbf{X}_t)$ are values of living in the home state and neighboring state respectively. One can show that there exists a critical value $\bar{\kappa}_{i,t} \equiv V_{-i}(\mathbf{X}_t) -$

[21] I found this setting gives stability to the solution, and it is reasonable in the sense that migration is a time-consuming process (so that it does not respond to fiscal policy changes immediately).

$V_i(\mathbf{X}_t)$ such that the migration decision is characterized by:

$$\begin{cases} m = 0 & \text{if } \kappa > \bar{\kappa}_{i,t}, \\ m = 1 & \text{if } \kappa \leq \bar{\kappa}_{i,t}. \end{cases}$$

As the result, the after-migration population of state i equals:

$$n_{i,t} = \left[1 - \Phi_{i,t}\left(\bar{\kappa}_{i,t}\right)\right] n_{i,t-1} + \Phi_{-i,t}\left(-\bar{\kappa}_{i,t}\right) n_{-i,t-1}. \tag{1.1}$$

$\Phi_{i,t}(\cdot)$ is the C.D.F. of κ which follows:

$$\Phi_{i,t}(\kappa) = \frac{1}{1 + exp\left(-\frac{\kappa - (\mu_\kappa - \beta_\kappa \log z_{i,t})}{\sigma_\kappa}\right)}.$$

The migration shock is drawn from a Logistic Distribution with mean $\mu_\kappa - \beta_\kappa \log z_{i,t}$ and standard deviation σ_κ. Having the mean be decreasing in $z_{i,t}$ captures the idea that the correlation between worker outflow and inflow is positive at the state-level.

After migration has stopped, a household choosing to live in state i faces the following problem:

$$\max_{c_{i,t}, h_{i,t}} U\left(c_{i,t}, h_{i,t}, g_{i,t} n_{i,t}^{-\eta}\right)$$

subject to:

$$\left(1 - \tau_{p,i,t} - o_{p,t}\right) w_{i,t} h_{i,t} + \frac{D_t}{\bar{n}} - c_{i,t} - \bar{\tau} = 0.$$

$g_{i,t} n_{i,t}^{-\eta}$ is the amount of local public goods. It equals to the *congestion-adjusted government expenditure*.[22] If $\eta = 1$, then the goods is completely rival. If $\eta = 0$,

[22] I find that including congestion gives stabilizes the solution.

it is completely non-rival. $\bar{\tau}$ represents taxes imposed by the lower level governments, hence it is not in the state government's choice set. $\tau_{p,i,t}$ and $o_{p,t}$ are the state and federal personal income tax rates respectively. The term $\frac{D_t}{\bar{n}}$ is dividend from the investor. The implicit assumption is that each household owns a share of the total asset.

In this paper, I use labor income tax as a proxy of personal income tax. This is because wages and salaries are the most important income source of households. In reality, most capital income concentrates in the higher income class. In this model, however, dividend is evenly distributed. Therefore, including dividend into the tax base tends to overstate the importance of this income in the determination of optimal personal tax rate and complicates the problem.

Following the standard assumptions, the utility function satisfies $U_c > 0$, $U_{cc} < 0$, $U_h < 0$, $U_{hh} < 0$, $U_{ch} > 0$ and $U_g > 0$.

Solving this problem, the trade-off between labor and consumption is characterized by:

$$(1 - \tau_{p,i,t} - o_{p,t})\, w_{i,t} = -\frac{U_{h,i,t}}{U_{c,i,t}}.$$

Plug it into the budget constraint, one could have:

$$-\frac{U_{h,i,t}}{U_{c,i,t}} h_{i,t} + \frac{D_t}{\bar{n}} - c_t - \bar{\tau} = 0. \tag{1.2}$$

Firm

The firm locates in state i produces output with capital and labor. The neoclassical production function $F\left(k_{i,t}, l_{i,t}, z_{i,t}\right)$ satisfies:

$$y = F_k k + F_l l.$$

The state-specific total factor productivity (TFP) $z_{i,t}$ follows

$$\log z_{i,t} = (1-\rho_z)\log(z_i^{s.s.}) + \rho_z \log z_{i,t-1} + \sigma_z \varepsilon_{z,i,t}. \tag{1.3}$$

$z_i^{s.s.}$ is the state-specific steady state TFP,[23] $\varepsilon_{z,i,t}$ is drawn from a standard Normal distribution.

The profit maximization problem reads as

$$\max_{k_{i,t}, l_{i,t}} \left(1 - o_{k,t} - \tau_{k,i,t}\right)\left[F\left(k_{i,t}, l_{i,t}, z_{i,t}\right) - w_{i,t} l_{i,t} - \delta k_{i,t}\right] - r_t k_{i,t}.$$

$\tau_{k,i,t}$ and $o_{k,t}$ denote the state and federal corporate income tax rates respectively. The taxable income equals output net of employee compensation and capital depreciation. One can write the factor demand functions as

$$r_t = \left(1 - o_{k,t} - \tau_{k,i,t}\right)\left(F_{k,i,t} - \delta\right),$$

$$w_{i,t} = F_{l,i,t}.$$

In the equilibrium, labor market clearing requires $l_{i,t} = n_{i,t} h_{i,t}$.

[23]In the baseline calibration $z_i^{s.s}$ is set to 1, because this paper wants to focus on fiscal competition itself, rather than the interaction of state heterogeneity and fiscal competition.

Investor

The nationwide investor in this economy derives utility from the dividends she pays to the households. The optimization problem reads as:

$$\max_{\{D_t, K_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \tilde{U}\left(D_t\right)$$

subject to:

$$(1 + r_t)A_t = A_{t+1} + D_t.$$

In this economy, asset contains capital and government bonds such that:

$$A_t = K_t + \sum_i b_{i,t}. \tag{1.4}$$

K_t is the total capital stock and $b_{i,t}$ is the government bond in state i.

The Euler equation for the investor is

$$\tilde{U}'\left(D_t\right) = \beta E_t\left[(1 + r_{t+1})\tilde{U}'\left(D_{t+1}\right)\right].$$

The solution of this problem can be written as:

$$A_{t+1} = G\left(r_t, \mathbf{X}_t\right), \tag{1.5}$$

$$D_t = (1 + r_t)A_t - G\left(r_t, \mathbf{X}_t\right). \tag{1.6}$$

Where $\mathbf{X}_t$ is the state variables in period t.

Federal Government

I put my attentions on state fiscal policies and the strategic reaction of the federal government is not target of interests. Therefore, I assume $o_{k,t}$ and $o_{p,t}$ follow the following processes:

$$o_{k,t} = (1-\rho_k)o_k^{ss} + \rho_k o_{k,t-1} + \sigma_k \varepsilon_{k,t}, \tag{1.7}$$

$$o_{p,t} = (1-\rho_p)o_p^{ss} + \rho_p o_{p,t-1} + \sigma_p \varepsilon_{p,t}. \tag{1.8}$$

ε_k and ε_p are drawn from standard Normal distributions.

State Government

The benevolent state government i finances public goods through issuing one-period bond and taxing production factors under source principal. While making decisions, the government takes its impact on the capital market into account (see appendix for more details). Let $\iota_{i,t}$ denote the overall corporate tax rate such that $\iota_{i,t} \equiv o_{k,t} + \tau_{k,i,t}$, the amount of capital used in state i can be written as:

$$k_{i,t} = K\left(\iota_{i,t}, \iota_{-i,t}, l_{i,t}, l_{-i,t}, z_{i,t}, z_{-i,t}, K_t\right), \tag{1.9}$$

$$r_t = R\left(\iota_{i,t}, \iota_{-i,t}, l_{i,t}, l_{-i,t}, z_{i,t}, z_{-i,t}, K_t\right). \tag{1.10}$$

Using the factor demand functions, the period-budget constraint of government i is:

$$g_{i,t} + (1+r_t)b_{i,t} + \Gamma(b_{i,t+1}) = \tau_{k,i,t}\left(F_{k,i,t} - \delta\right)k_{i,t} + \tau_{p,i,t}F_{l,i,t}l_{i,t} + b_{i,t+1} + n_{i,t}\bar{\tau}.$$

In which $\Gamma(b_{i,t+1})$ is the cost of bond issuance.[24] Following the standard transformation, one can rewrite the government budget as:

$$F\left(k_{i,t}, l_{i,t}, z_{i,t}\right) - \delta k_{i,t} + b_{i,t+1} + n_{i,t}\bar{\tau} - o_{k,t}\left(F_{k,i,t} - \delta\right) k_{i,t} - o_{p,t} F_{l,i,t} l_{i,t}$$

$$= g_{i,t} + (1 + r_t) b_{i,t} + r_t k_{i,t} - \frac{U_{h,i,t}}{U_{c,i,t}} n_{i,t} h_{i,t} + \Gamma(b_{i,t+1}). \tag{1.11}$$

The term $-\frac{U_{h,i,t}}{U_{c,i,t}} n_{i,t} h_{i,t}$ comes from the labor-consumption trade-off condition that $(1 - o_{p,t} - \tau_{p,i,t}) w_{i,t} = -\frac{U_{h,i,t}}{U_{c,i,t}}$.

The Markov Perfect Equilibrium

Let $\pi_{i,t} \equiv [c_{i,t},\ h_{i,t},\ g_{i,t},\ \tau_{k,i,t},\ b_{i,t+1}]$, the Ramsey problem of state government i can be written as:

$$V_{i,t} = \max_{\pi_{i,t}, K_{t+1}} U\left(c_{i,t}, h_{i,t}, g_{i,t} n_{i,t}^{-\eta}\right)$$

$$+\beta E_t \left[\left[1 - \Phi_{i,t+1}\left(\bar{\kappa}_{i,t+1}\right)\right] V_{i,t+1} + \int_{-\infty}^{\bar{\kappa}_{i,t+1}} \left(V_{-i,t+1} - \kappa\right) d\Phi_{i,t+1}(\kappa) \right],$$

subject to:

$$\text{equation } (1.1) - (1.11).$$

Let $\mathbf{X}_t$ denote the vector of state variables of period t such that

$$\mathbf{X}_t \equiv \left[n_{i,t},\ z_{i,t},\ z_{-i,t},\ b_{i,t},\ b_{-i,t},\ o_{k,t},\ o_{l,t},\ A_t\right].$$

[24] In the U.S., the requirement of maintaining a balanced operating budget limits the ability of using government bond to finance current expenditure. However, as mentioned by Maguire (2011), the limitation might be not as hard as anticipated. First, many states create special purpose authorities, which are not restricted by the requirement, for borrowing. Second, most states run capital budgets to finance project such as infrastructure, in which government bond is an important component and the interest payment is managed by the operating budgets. Some states are even flexible to shift expenditure between the operating and capital budget. Also notice that to simplify the analysis, I assume there is only one type of public good, which can be loosely viewed as a mixture of government service and public capital. Therefore, in this model the state government can issue bond, but has to pay a cost denoted by $\Gamma(b)$.

The equilibrium is defined below.

Definition: The Markov perfect equilibrium is defined as: i) the value functions: $V_i(\mathbf{X}_t)$ and $V_{-i}(\mathbf{X}_t)$; ii) the policy functions: $\Pi_i(\mathbf{X}_t)$ and $\Pi_{-i}(\mathbf{X}_t)$; iii) the law of motion for populations: $N_i(\mathbf{X}_t)$ and $N_{-i}(\mathbf{X}_t)$; iv) the law of motion for asset: $\hat{G}(\mathbf{X}_t)$. Such that:

1. For any state i, given $n_{i,t} = N_i(\mathbf{X}_t)$ and $n_{-i,t} = N_{-i}(\mathbf{X}_t)$, functions $\Pi_i(\cdot)$, $\hat{G}(\cdot)$, $V_i(\cdot)$, and $V_{-i}(\cdot)$ solve the local Ramsey problem taking $\pi_{-i,t} = \Pi_{-i}(\mathbf{X}_t)$ as given;

2. Given $V_i(\cdot)$ and $V_{-i}(\cdot)$, $N_i(\cdot)$ and $N_{-i}(\cdot)$ are consistent with equation (1.1);

3. $\hat{G}(\cdot)$ satisfies equation (1.5) such that $\hat{G}(\mathbf{X}_t) = G(r(\mathbf{X}_t), \mathbf{X}_t)$.

1.3.2 The Optimal State Policies

To have a clear picture of the optimal state fiscal policies, I assume the utility function takes the following form:

$$\log\left(c - \frac{h^{1+1/\varphi}}{1+1/\varphi}\right) + \chi \log\left(gn^{-\eta}\right).$$

The Greenwood–Hercowitz–Huffman (GHH) functional form of U suggests that the income effect of labor supply is eliminated, which simplifies the problem. As a result, the optimal hours worked satisfies $h = [(1 - \tau_p - o_p) w]^{\varphi}$ and φ is the labor supply elasticity.

Assume the investor's utility function $\tilde{U}(D_t) = \log(D_t)$, therefore $G(r_t, \mathbf{X}_t)$ reduces to $\equiv \beta(1 + r_t)A_t$ as the income and substitution effect offset each other.

Additionally, assume $F(k,l,z) = zk^{\alpha}l^{1-\alpha}$, hence the transformed government budget becomes:

$$(1-f_t)\, z_{i,t}k_{i,t}^{\alpha}l_{i,t}^{1-\alpha} - \delta\,(1-o_{k,t})\, k_{i,t} + n_{i,t}\bar{\tau} + b_{i,t+1}$$

$$-g_{i,t} - (1+r_t)b_{i,t} - n_{i,t}h_{i,t}^{1+1/\varphi} - r_t k_{i,t} - \Gamma(b_{i,t+1}) = 0. \tag{1.11a}$$

f_t is the weighed average of $o_{k,t}$ and $o_{p,t}$ such that $f_t \equiv \alpha o_{k,t} + (1-\alpha)o_{p,t}$. From (1.9a), the variations of f_t and $z_{i,t}$ have similar impacts on the economy.

While solving the local Ramsey problem, let $\psi_{i,t}$ be the multiplier associated with the modified household budget; let $\mu_{i,t}$ be the multiplier associated with the government budget; let $\theta_{i,t}$ be the multiplier associated with the law of motion for A. By construction, $\psi_{i,t}$ is the shadow price of household consumption; $\mu_{i,t}$ is the shadow price of government expenditure; and $\theta_{i,t}$ is the shadow price of the future assets.

The Optimal State Corporate Income Tax Rate

The optimal $\tau_{k,i,t}$ is given by:

$$\tau_{k,i,t} = \frac{\frac{\partial r_t}{\partial \tau_{k,i,t}}\left(k_{i,t} + b_{i,t} - \frac{\beta\theta_{i,t}+(1-\beta)\psi_{i,t}/\bar{n}}{\mu_{i,t}}A_t\right)}{\frac{\partial k_{i,t}}{\partial \tau_{k,i,t}}(F_{k,i,t}-\delta)} + \frac{(1-\alpha)\,(o_{p,t}-o_{k,t})}{1-\frac{\delta}{F_{k,i,t}}}. \tag{1.12}$$

Equation (1.12) comes the necessary condition for $\tau_{k,i,t}$. The first term represents the force from horizontal competition. A marginal increase in $\tau_{k,i,t}$ generates a “price effect” by lowering the interest rate (the numerator). This effect lowers the cost of capital and bond payment (which relaxes the budget constraint), but also lowers the household income and the future assets. Therefore the optimal tax rate depends on the net of these two changes. Second, a marginal increase in $\tau_{k,i,t}$ also generates an "allocation effect" by decreasing the amount of capital in the home state (the denominator). This effect shrinks the tax base of corporate income tax, therefore the optimal tax rate depends on the sensitivity of tax base with respect to the tax rate.

Ceteris paribus, consider the case that the neighboring state increases $\tau_{k,-i,t}$ or $\tau_{p,-i,t}$. Such a change causes capital to flow into the home state (recall that higher $\tau_{p,-i,t}$ lowers the neighboring state's labor supply, therefore also affects the allocation of capital). The inflow of capital in the home state strengthens the price effect (because the cost of capital becomes higher) and weakens the allocation effect (because it lowers $F_{k,i,t}$ and makes the tax base insensitive with respect to $\tau_{k,i,t}$). As a result, the home state will react by raising $\tau_{k,i,t}$.

The second term of (1.12) represents the force from vertical competition. To have a clear intuition, first consider the marginal effect of capital on a state's budget constraint. A marginal increase in $k_{i,t}$ changes the constraint by $\tau_{k,i,t}(F_{k,i,t} - \delta) - o_{k,t}F_{kk,i,t}k_{i,t} - o_{p,t}F_{kl,i,t}l_{i,t}$ units.[25] As can be seen, the marginal effect of capital is increasing in $o_{k,t}$ but decreasing in $o_{p,t}$. This is because more capital decreases (increases) the federal corporate (personal) income tax by attenuating (expanding) the tax base. As a result, an increase in $o_{k,t}$ provides the state planner an incentive to attract more capital by lowering $\tau_{k,i,t}$; an increase in $o_{p,t}$ provides the incentive to reduce capital by raising $\tau_{k,i,t}$. These reactions are consistent with the empirical findings above.

[25]The first term comes from $F_{k,i,t} - \delta - r_t - o_{k,t}(F_{k,i,t} - \delta)$.

The Optimal State Personal Income Tax Rate

The optimal $\tau_{p,i,t}$ is given by:

$$\tau_{p,i,t} = \frac{1 + \varphi f_t - \frac{\psi_{i,t}}{n_{i,t}\mu_{i,t}}}{1 + \varphi - \frac{\psi_{i,t}}{n_{i,t}\mu_{i,t}}} - o_{p,t}. \tag{1.13}$$

Equation (1.13) comes from the necessary condition for $h_{i,t}$. As can be seen, an important determinant of $\tau_{p,i,t}$ is $\frac{\psi_{i,t}}{n_{i,t}\mu_{i,t}}$, which measures the marginal rate of substitution (MRS) of household consumption for public goods because it is the ratio of shadow prices.[26] By definition, a higher MRS implies that the state planner is willing to sacrifice more government expenditure to gain one more unit of household consumption, hence $\tau_{p,i,t}$ should decrease.

Ceteris paribus, consider the case that the neighboring state increases $\tau_{k,-i,t}$ or $\tau_{p,-i,t}$. As described above, such a change re-allocates capital towards the home state. In the home state, the inflow of capital relaxes the budget constraint and lowers the shadow price of government expenditure, hence increases the MRS. As a result, the home state will react by decreasing $\tau_{p,i,t}$.

Similar to $\tau_{k,i,t}$, federal tax rates also influence $\tau_{p,i,t}$. The marginal effect of labor on a state's budget constraint equals $\tau_{p,i,t}F_{l,i,t} - o_{k,t}F_{kl,i,t}k_{i,t} - o_{p,t}F_{ll,i,t}l_{i,t}$,[27] which is decreasing in $o_{k,t}$ and increasing in $o_{p,t}$. This is because having more labor increases (decreases) the federal corporate (personal) income tax by expanding (attenuating) the tax base. As a result, an increase in $o_{k,t}$ provides the state planner an incentive to discourage the labor supply by increasing $\tau_{p,i,t}$; an increase in $o_{p,t}$ provides the incentive to encourage labor supply by decreasing

[26] Since the g is non-rival, the denominator should time n.
[27] The first term comes from $F_{l,i,t} - (1 - o_{p,t} - \tau_{p,i,t})\, w_{i,t} - o_{p,t}F_{l,i,t}$.

$\tau_{p,i,t}$. These reactions are consistent with the empirical findings.

The law of motion for μ and θ

The shadow prices of public goods $\mu_{i,t}$ and future capital $\theta_{i,t}$ move forward according to the following rules:

$$\mu_{i,t}\left[1-\Gamma'(b_{i,t+1})\right]-\theta_{i,t}=-\beta E_t\left[\left(1-\Phi_{i,t+1}\left(\bar{\kappa}_{i,t+1}\right)\right)\frac{\partial V_{i,t+1}}{\partial b_{i,t+1}}+\Phi_{i,t+1}\left(\bar{\kappa}_{i,t+1}\right)\frac{\partial V_{-i,t+1}}{\partial b_{i,t+1}}\right], \tag{1.14}$$

$$\theta_{i,t}=\beta E_t\left[\left(1-\Phi_{i,t+1}\left(\bar{\kappa}_{i,t+1}\right)\right)\frac{\partial V_{i,t+1}}{\partial A_{t+1}}+\Phi_{i,t+1}\left(\bar{\kappa}_{i,t+1}\right)\frac{\partial V_{-i,t+1}}{\partial A_{t+1}}\right]. \tag{1.15}$$

The first equation comes from the necessary condition for $b_{i,t+1}$. The left-hand-side is the gain of issuing one more unit of government bond, which equals the marginal welfare brought by the extra units of government expenditure net of the crowd-out effect. The future marginal impact of bond issuance (the right-hand-side) is composed of two parts: i) the welfare of living in state i changes by $\frac{\partial V_{i,t+1}}{\partial b_{i,t+1}}$ and it affects $1-\Phi_{i,t+1}\left(\bar{\kappa}_{i,t+1}\right)$ of current residents; ii) the welfare of living in state $-i$ changes by $\frac{\partial V_{-i,t+1}}{\partial b_{i,t+1}}$, which, because of migration, affects $\Phi_{i,t+1}\left(\bar{\kappa}_{i,t+1}\right)$ of current residents.

It is meaningful to discuss the relationship between migration and borrowing. Compared with the Euler equation where households are immobile:

$$\mu_{i,t}\left[1-\Gamma'(b_{i,t+1})\right]-\theta_{i,t}=-\beta E_t\frac{\partial V_{i,t+1}}{\partial b_{i,t+1}},$$

migration brings some new features.

Firstly, migration makes a state government partially altruistic when deciding the amount of bond issuance. By construction, some households will out-migrate to the neighboring state in the next period and their welfare is also included in the state planner's object. As a result, the marginal effect of $b_{i,t+1}$ on $V_{-i,t+1}$ also exists in the right-hand-side. Such a structure increases the incentive of borrow because only $1-\Phi_{i,t+1}\left(\bar{\kappa}_{i,t+1}\right)$ of current residents have to bear the burden, and the other $\Phi_{i,t+1}\left(\bar{\kappa}_{i,t+1}\right)$ of current residents will benefit from raising $b_{i,t+1}$ when $\frac{\partial V_{-i,t+1}}{\partial b_{i,t+1}}>0$.

Secondly, the effects of $b_{i,t+1}$ on $V_{i,t+1}$ and $V_{-i,t+1}$ are different from the immobile-household case. Specifically, denote ζ_n the marginal effect of n on the a state's budget constraint, one can show that $\frac{\partial V_{i,t+1}}{\partial b_{i,t+1}}$ equals to $-\mu_{i,t+1}+\frac{\partial n_{i,t+1}}{\partial b_{i,t+1}}\left(\mu_{i,t+1}\zeta_{n,i,t+1}-\frac{\chi\eta}{n_{i,t+1}}\right)\neq-\mu_{i,t+1}$. The reason is that higher $b_{i,t+1}$ implies a tighter budget constraint in the next period; knowing that the tax burden will be heavier, more households will move out of the home state, which further tightens the government budget and eases its congestion. Meanwhile, as more households move into state $-i$, the higher $n_{-i,t+1}$ will relax the neighboring state's budget constraint and intensify its congestion: $\frac{\partial V_{-i,t+1}}{\partial b_{i,t+1}}$ equals to $\frac{\partial n_{-i,t+1}}{\partial b_{i,t+1}}\left(\mu_{-i,t+1}\zeta_{n,-i,t+1}-\frac{\chi\eta}{n_{-i,t+1}}\right)\neq 0$, the sign is positive when $\chi\eta$ is sufficiently small.

Finally, equation (1.15) is the necessary condition for A_{t+1}. Similar to (1.14), equation (1.15) implies that the state planner should include the effect of A_{t+1} on both $V_{i,t+1}$ and $V_{-i,t+1}$ into consideration.

1.4 Quantitative Analysis

This section calibrates the model to match the state-level moments in the U.S., and then analyzes the effect of a federal tax rate shock quantitatively. The previous section has assigned most of functional forms except the cost of bond issuance. In this section, assume the cost of takes the form $\Gamma(b) = \frac{B}{2}b^2$.

1.4.1 Calibration

The unit of time is set to a year, which replicates the fact that state governments set policies every fiscal year. I assign $\beta = 0.96$, $\delta = 0.1$, $\alpha = 0.36$, and $\varphi = 0.5$. These values are widely assigned in macroeconomics literature.

FRED reports the U.S. annual TFP, I assume the state-level TFP has the same persistence and standard deviation,[28] $\rho_z = 0.8733$ and $\sigma_z = 0.01$. The persistence of o_k and o_l are computed on the basis of the NIPA data set from 1987 to 2017, $\rho_k = 0.8354$ and $\rho_p = 0.7396$. The standard deviations of the shocks are re-scaled by the calibrated o_k^{ss} and o_p^{ss}, $\sigma_k = 0.0136$ and $\sigma_p = 0.0049$.

The rest of the parameters include $\left[\mu_\kappa,\ \sigma_\kappa,\ \chi,\ \bar{\tau},\ o_k^{ss},\ o_p^{ss},\ \beta_\kappa,\ \eta,\ B\right]$. They are calibrated to match the following moments: (1) the steady state out-migration rate = 0.03; (2) the steady state government bond as a fraction of output = 0.14; (3) the steady state government expenditure as a fraction of output = 0.1; (4) the lump-sum tax as a fraction of state government expenditure = 0.7; (5) the size of state personal tax relative to corporate tax = 6.3457; (6) the steady state federal tax revenue as a fraction of output = 0.1; (7) the correlation between

[28]I make this assumption because the focus is fiscal competition itself, rather than the interaction of inter-state heterogeneity and competition.

Table 1.2: Parameter Values

Parameter	Description	Value
β	discount factor	0.96
φ	labor elasticity	0.5
α	capital share	0.36
δ	depreciation rate	0.1
ρ_z	persistence of z	0.8733
σ_z	std deviation of z shock	0.01
ρ_k	persistence of o_k	0.8354
ρ_p	persistence of o_l	0.7396
σ_k	std deviation of o_k shock	0.0136
σ_p	std deviation of o_l shock	0.0049
μ_κ	mean of κ: constant	1.738
β_κ	mean of κ: coefficient	0.883
σ_κ	standard deviation of κ	0.5
χ	taste of public goods	0.2702
$\bar{\tau}$	lump-sum tax	0.1059
o_k^{ss}	s.s. federal corp. tax rate	0.1179
o_p^{ss}	s.s. federal pers. tax rate	0.1342
η	degree of public goods congestion	0.3
B	size of bond issuance cost	0.1255

worker outflow and inflow = 0.63; (8) the regression coefficient of $\log(g)$ on $\log(n)$ = 0.76; (9) the standard deviation of bond-to-output ratio = 0.04. Table 1.2 displays all the parameter values.

1.4.2 Steady State Results

The parameter values suggest $\tau_k^{ss} = 0.0437$ and $\tau_p^{ss} = 0.0518$. The value of o_k^{ss} and o_p^{ss} implies the ratio of federal personal to corporate income tax is 6.09 while the number in data is 4.66. Since the model assumes the federal government only impose corporate and personal income taxes in this model, the numbers are comparable.

To follow up with the discussion of how labor mobility affects fiscal policies, table 1.3 compares several selected steady state values under different values of μ_κ.

Other parameters fixed, lowering μ_κ increases the out-migration rate, which leads to a higher incentive to borrow as discussed above. In the steady state, the higher bond issuance tightens a state's budget constraint and decreases government expenditure. The tightening budget decreases the MRS of household consumption for public expenditure, therefore increases the state personal income tax rate. Besides, the higher bond issuance also motivates the state planner to lower the interest rate, therefore increases the state corporate tax rate. Having higher tax rates on production factors, the steady state output is lower.

Table 1.3: The Effect of Labor Mobility

μ_κ	out-rate	b^{ss}	g^{ss}	τ_k^{ss}	τ_p^{ss}	y^{ss}
1.7380	0.0300	0.2270	0.1513	0.0437	0.0518	1.5133
1.6380	0.0364	0.2761	0.1507	0.0444	0.0548	1.5101
1.5380	0.0441	0.3350	0.1497	0.0453	0.0588	1.5060
1.4380	0.0534	0.4056	0.1485	0.0465	0.0639	1.5006
1.3380	0.0644	0.4897	0.1467	0.0482	0.0707	1.4984

To study the effect of horizontal competition, it is meaningful to compare the steady state results with and without factor mobility. To implement this exercise, I construct an economy in which both factors are immobile (I call it the IF model hereafter). Qualitatively speaking, this alternative model looks as if there is only one state (see appendix for the necessary conditions). The two models have the same settings except for factor mobility and congestion. Table 1.4 displays the result.

Table 1.4: The Effect of Horizontal Competition – Steady State Results

	τ_k^{ss}	τ_p^{ss}	y^{ss}	c^{ss}	h^{ss}	K^{ss}	b^{ss}
Baseline Model	0.0437	0.0518	1.5133	0.8435	0.9238	7.2783	0.2270
IF Model	0.0540	0.0380	1.5208	0.8511	0.9305	7.2842	-0.0088

Compared with the baseline model, taking away factor mobility increases τ_k^{ss} and decreases τ_p^{ss}. The reason is that capital becomes completely predetermined and inelastic to τ_k, but labor is still sensitive to τ_p as it affects the after-tax wage. Besides, the absence of labor mobility reduces the state planner's borrowing incentive and results in a small budget surplus.

Interestingly, factor mobility leads to a smaller output in the steady state. Although τ_k^{ss} is lower in the baseline model, the higher bond issuance crowds out capital investment and lowers K^{ss}. Also, the lower K^{ss} and higher τ_p^{ss} reduces the after-tax wage relative to the alternative model and discourages the labor supply.

1.4.3 The Effect of Federal Tax Rate Shocks

This section studies the effects of federal tax rate shocks on the economy and analyzes the influence of fiscal competition.

For each shock, I compute the impulse responses in three different models. The invariant-state-policy (hear after ISP) model , which fixes the state fiscal policies at the baseline model's steady-state levels and excludes both vertical and horizontal competitions. The immobile-factor (hereafter IF) model, which endogenizes the state fiscal policies but assumes factors are immobile, hence only considers vertical competition. The baseline model, which takes both types of fiscal competition into account.

Comparing the impulse response of output in the three models provides valuable information. The difference between the ISP model and the IF model shows the effect of *direct vertical competition* — vertical competition without the influence of factor mobility; the difference between the IF model and the baseline model tells the role of horizontal competition; the difference between the ISP model and the baseline model displays the net influence of state fiscal policies' responses.

Intuition

At this stage, it is helpful to illustrate how horizontal competition shapes a state's responses of tax rates to a federal shock. Let x and y be different types of tax. For simplicity, let τ^* denote tax rate in the neighboring state. In a simple static framework, one can write $\tau_x = T\left(o_k, o_p, \tau_x^*, \tau_y^*\right)$. Taking derivatives with respect to o_k and o_p:

$$\frac{d\tau_x}{do_k} = T_1 + T_3\frac{d\tau_x^*}{do_k} + T_4\frac{d\tau_y^*}{do_k}; \quad \frac{d\tau_x}{do_p} = T_2 + T_3\frac{d\tau_x^*}{do_p} + T_4\frac{d\tau_y^*}{do_p}.$$

In the equations above, T_1 and T_2 measure the direct effects of federal tax rates on state tax rate x, which are the effects represented by the IF model. T_3 and T_4 measure the interdependence of tax rates in different states, they are non-zero because of horizontal competition. In the symmetric equilibrium,

$$\frac{d\tau_x}{do_k} = \frac{T_1}{1-T_3} + \frac{T_4\frac{d\tau_y^*}{do_k}}{1-T_3}; \quad \frac{d\tau_x}{do_p} = \frac{T_2}{1-T_3} + \frac{T_4\frac{d\tau_y^*}{do_p}}{1-T_3}.$$

Compare the actual response with the direct effect, one will have:

$$\frac{d\tau_x/do_k}{T_1} - 1 = \frac{T_3}{1-T_3} + \frac{\frac{T_4}{T_1}\frac{d\tau_y^*}{do_k}}{1-T_3}; \quad \frac{d\tau_x/do_p}{T_2} - 1 = \frac{T_3}{1-T_3} + \frac{\frac{T_4}{T_2}\frac{d\tau_y^*}{do_p}}{1-T_3}.$$

First, horizontal competition generates an amplification (dampening) effect when T_3 is positive (negative). The reason is that since τ_x^* responds to o_k and o_p, τ_x will react to the resulting change in τ_x^*. Second, horizontal competition also creates a cross-tax effect. The reason is that since τ_y^* responds to o_k and o_p, τ_x will react to the resulting change in τ_y^*. Crucially, the sign of the cross-tax effect depends on the specific type of x. One can show that if x represents corporate income tax, the cross-tax effect is negative, which also goes against the amplification effect. Instead, if x represents personal income tax, the cross-tax effect is positive, which goes against the dampening effect. As a result, the effect of horizontal competition is ambiguous.

Federal Corporate Tax Rate Cut

Assume the economy is initially at the steady state. This part studies the effect of $\varepsilon_k = -1\%$. Figure 1.3 displays the impulse response functions in all the three models.

The blue-dashed lines display the responses in the ISP model. After the shock takes place, in the left-most graph of the second row, output changes 0% on the impact because capital is predetermined and labor does not respond due to the invariant state tax rates. In the coming periods, output gradually increases as the lower tax rate on corporate income encourages capital investment and labor supply (because the capital-labor ratio is higher). Therefore, there exists a hump

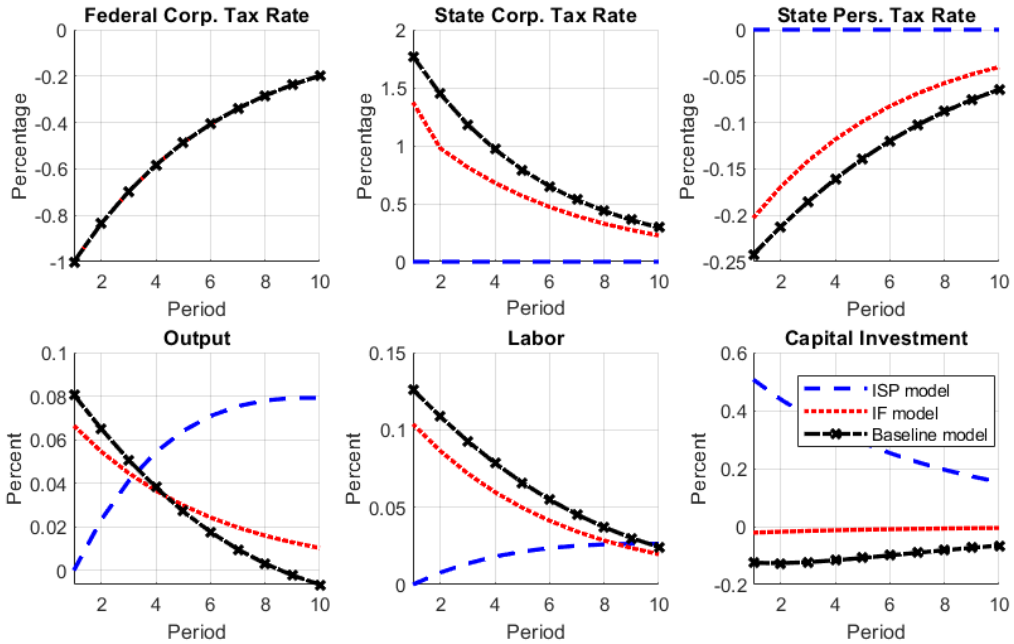

Figure 1.3: IRFs to Federal Corp. Tax Rate Cut

shape in the impulse response, the peak (0.08%) comes in period 9.

The red-dotted lines depict the impulse responses in the IF model. In response to the federal shock, output increases by 0.065% on the impact. This is because in the right-most graph of the first row, the state personal income tax rate decreases by 0.2% and it encourages labor supply. Instead of the hump shape, output falls monotonically in the following periods, its value decreases to 0.01% in the tenth period. The reason is that the over-response of state corporate income tax rate raises the overall tax rate on corporate income and (slightly) decreases capital investment. In the middle graph of the first row, τ_k increases by 1.34% on the impact and then gradually moves back to the steady state. This over-response lies in the setting that capital depreciation is deductible, which suggests the tax base of corporate income tax is $(F_k - \delta)$ instead of F_k for each unit of capital. As a result, the marginal benefit of attracting capital, $\tau_k \times (F_k - \delta)$, is small relative to its marginal cost, $o_p F_{kl} l + o_k F_{kk} k$, which gives the state planner an incentive to increase the magnitude of response.

The black-broken lines depict the impulse responses in the baseline model. On the impact, output increases by 0.08%, which is higher than that in the IF model. Since the cross-tax effect dominates the dampening effect, the response of τ_p decreases from -0.2% in the IF model to -0.24% on the impact. This additional decrease induces more labor supply and output. In the following periods, however, output falls faster relative to the IF model does, and the value even falls below the pre-shock level after nine periods. Since the amplification effect dominates the cross-tax effect, the increase in the state corporate tax rate is higher than that in the IF model. As one can see, τ_k increases by 1.77% while the value in the IF model is 1.34%. As a result, the capital investment decreases by more relative to the IF model therefore affects the response of output.

The Cumulative Effect

As discussed above, the responses of state fiscal policies influence the effect of a federal corporate tax rate cut on the economy. Since the relative magnitudes of output response in the three models change over time, it is meaningful to look at the cumulative effect.

Let $\hat{Y}_t$ be the percent deviation from the steady state level in period t, the cumulative effect of the shock on output up to period T can be calculated as:

$$CE(T) = \sum_{t=1}^{T} \beta^t \hat{Y}_t.$$

Figure 1.4 displays the cumulative effects of different models. When the economy goes back to the steady state, as can be seen, the responses of state fiscal

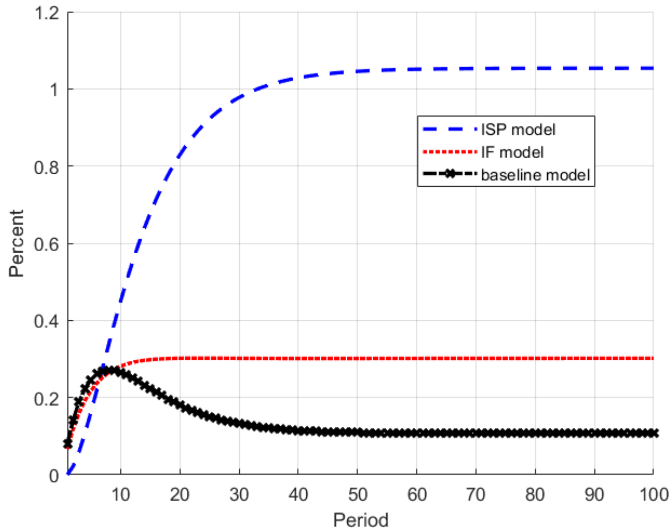

Figure 1.4: The Cumulative Effect of Federal Corp. Tax Cut

policies decrease the cumulative effect from 1.05% in the ISP model to 0.11% in the baseline model. Inside of this change, 0.75% (about 4/5) attributes to direct vertical competition, and 0.20% (about 1/5) attributes to horizontal competition.

Federal Personal Tax Rate Cut

Assume the economy is initially at the steady state. This part studies the effect of $\varepsilon_p = -1\%$. Figure 1.5 displays the impulse response functions in all the three models.

In response to the shock, output in the ISP model (the blue-dashed lines) increases by 0.33% on the impact because the lower rate on personal income encourages labor supply. After that, output falls towards the steady state as the shock fades.

In the IF model (the red-dotted lines) which only includes direct vertical competition, the federal shock results in an increase of 0.1% in output on the

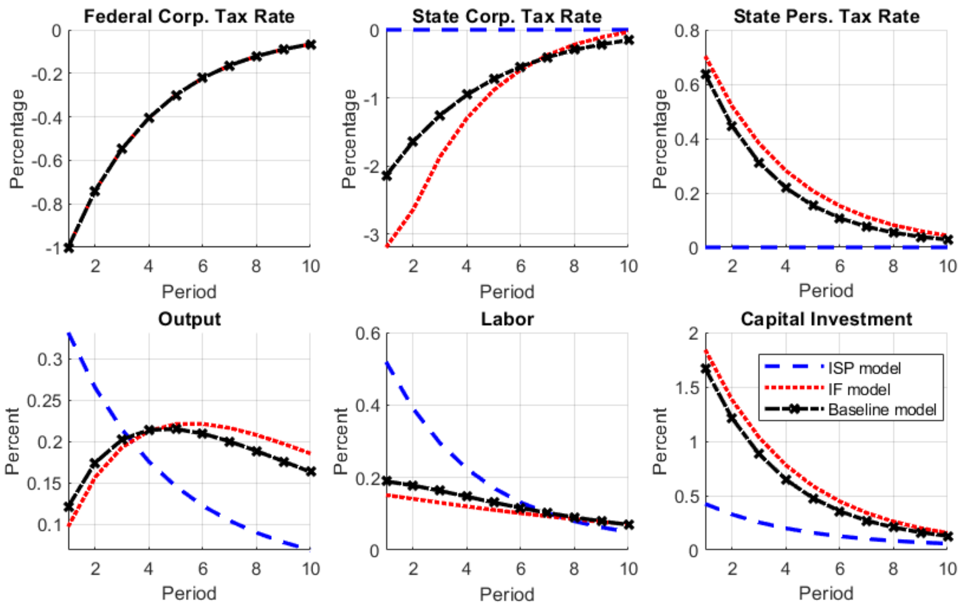

Figure 1.5: IRFs to Federal Pers. Tax Cut

impact. Relative to the ISP model, the force that moderates the initial output response is an increase of 0.70% in τ_p. Such an increase partially offsets the federal personal tax rate cut hence gives a smaller increase in labor supply and output. In the following periods, the response of output exhibits a hump shape and the peak (0.22%) happens in the sixth period. This is because the state corporate tax rate drops in response to the shock. In the middle graph of the first row, τ_k decreases by 3.20% on the impact and then gradually moves back to the steady state. As a result, the lower tax rate on corporate income stimulates capital investment and keeps increasing output in the following periods.

In the baseline model (the black-broken lines), output increases by 0.13% on the impact, which is higher relative to the value in the IF model (0.10%). This difference attributes to the negative inter-state correlation of state personal tax rate, which gives a dampening effect in the response of τ_p and dominates the cross-tax effect. As can be seen, the state personal tax rate increases by 0.63%

on the impact, and the corresponding value in the IF model is 0.70%. Having a bigger decrease in the overall tax rate on personal income, the increase in labor supply is higher, so is output. Similar to the IF model, the response of output also exhibits a hump shape in the baseline model. However, the peak comes earlier because the increase in capital investment is smaller. Since the cross-tax effect dominates the amplification effect, the state corporate tax rate decreases by less relative to the IF model. One the impact, τ_k drops by 2.14% instead of 3.20%.

The Cumulative Effect

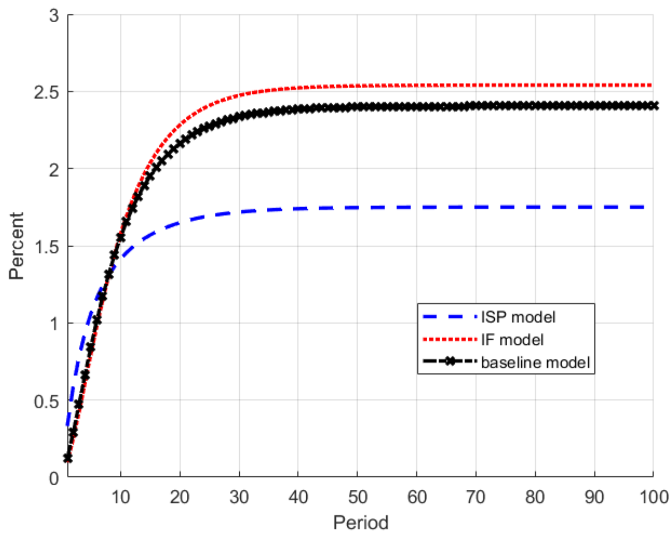

Figure 1.6: The Cumulative Effect of Federal Pers. Tax Cut

Figure 1.6 displays the cumulative effects of different models. When the economy goes back to the steady state, as can be seen, the responses of state fiscal policies increase the cumulative effect from 1.75% in the ISP model to 2.40% in the baseline model. Starting from the ISP model, introducing direct vertical competition enlarges the cumulative effect by 0.79%. On this basis, adding horizontal competition decreases the effect by 0.13%, around 1/5 of the total change.

Sensitivity Check

This part studies how labor mobility and inter-state heterogeneity affect the responses to a federal tax shock. To illustrate the driving forces, I use the case of cutting the federal corporate tax rate as an example for the rest of this part. The logic can be extended to cutting the federal personal tax rate.

Labor Mobility

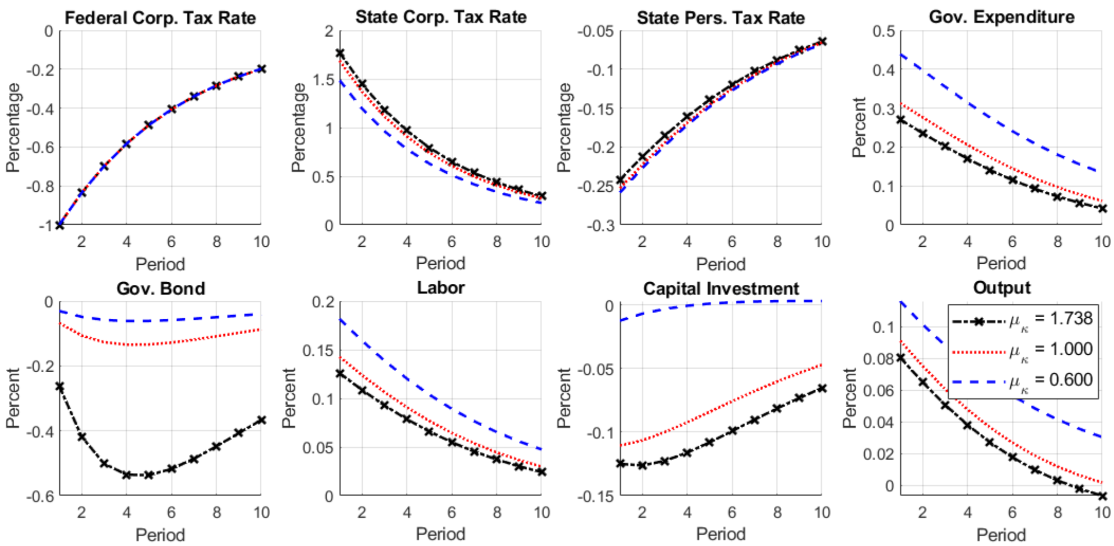

Figure 1.7: The Effect of Labor Mobility

Figure 1.7 displays the impulse responses to a federal corporate tax rate cut for different values of μ_κ (1.738, 1, and 0.6). Lowering μ_κ strengthens the borrowing incentive and intensifies the competition for labor. In response to a cut in the federal corporate tax rate, a model with a smaller μ_κ results in a smaller decrease in the bond issuance and a bigger increase in the government expenditure. Having a more relaxed budget constraint, the state planner decreases the personal tax rate by more. Moreover, the lower $\tau_{p,-i}$ leads to a stronger cross-tax effect and a smaller increase in the state corporate tax rate. As can be seen, the reactions of the state tax rates generate a bigger increase in labor supply and a smaller decrease in capital investment, which suggest that the response of output to the federal shock is bigger when labor is more mobile.

Inter-State Productivity Gap

Figure 1.8a and 1.8b display the impulse responses to a federal corporate tax rate cut for different gaps between $z_1^{s.s.}$ and $z_2^{s.s.}$. Specifically, let $z_1^{s.s.} + z_2^{s.s.} = 2$, I explore the cases that $z_1^{s.s.} - z_2^{s.s.} = 0$, 0.04, and 0.08 respectively. As can be seen from figure 1.8a, the productive state (state 1) benefits more from the federal tax cut and the gain is increasing in the inter-state productivity gap. As a result, a higher z gap is associated with a bigger reallocation of households towards state 1. Recall that $\tau_{p,i,t}$ also depends upon $\frac{\psi_{i,t}}{n_{i,t}\mu_{i,t}}$, the reaction of n_1 (n_2) implies that the decrease in $\tau_{p,1}$ ($\tau_{p,2}$) will be bigger (smaller) as the z gap goes wider. Moreover, due to the cross-tax effect, the response of $\tau_{k,1}$ ($\tau_{k,2}$) is decreasing in the gap. However, the magnitude of change is small.

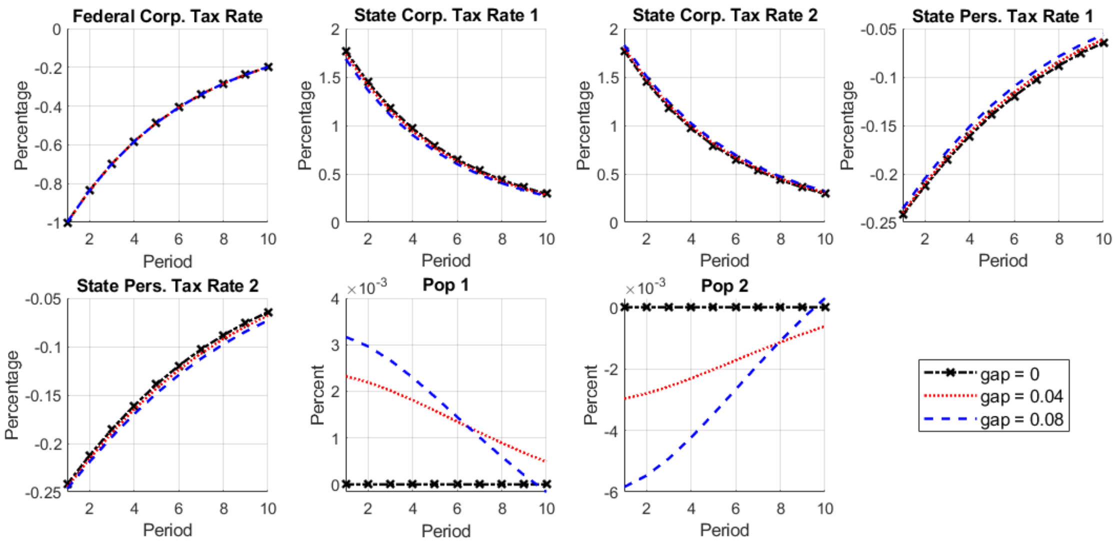

Figure 1.8a: The Effect of Inter-State Heterogeneity (1)

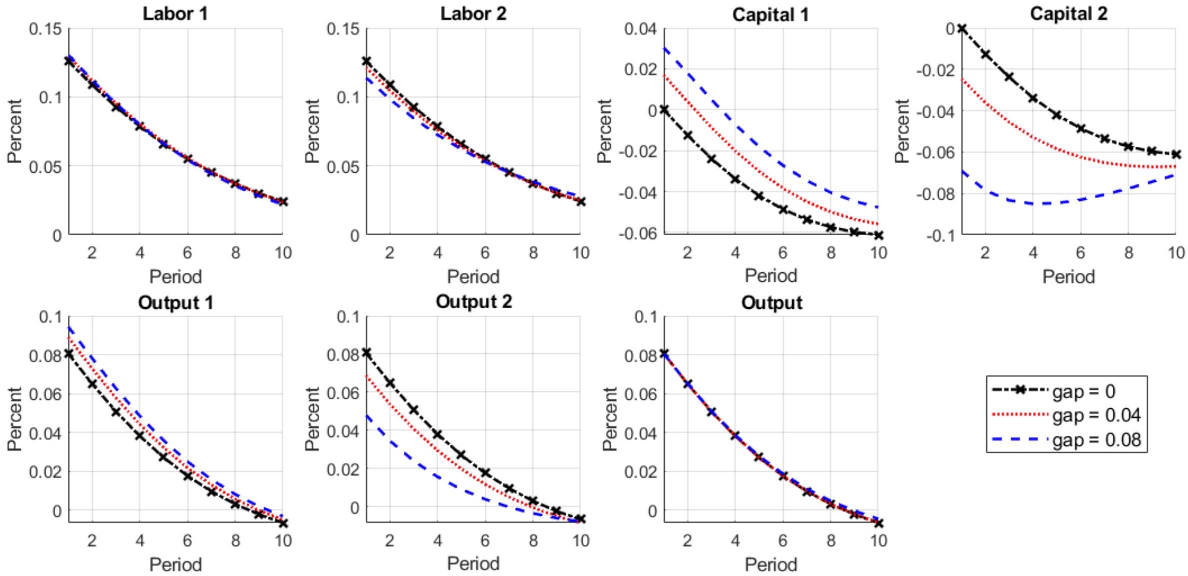

Figure 1.8b: The Effect of Inter-State Heterogeneity (2)

The first two graphs in figure 1.8a show that the responses of labor in the two states are almost invariant to the z gap. That is, the change in $\hat{n}_{i,t}$ is offset by the resulting changes in $\hat{F}_{l,i,t}$ and $\hat{\tau}_{p,i,t}$.[29] Although the responses of labor change little, the difference between $\hat{F}_{k,1}$ and $\hat{F}_{k,2}$ still goes bigger as the z gap increases. Therefore, in response to the federal shock, a larger value of $z_1^{ss} - z_2^{ss}$ is associated with a bigger capital reallocation towards state 1, which enlarges the gap between the output responses in state 1 and 2. On the aggregate level, however, the response of total output — $\hat{Y}_t = a\hat{y}_{1,t} + (1-a)\hat{y}_{2,t}$ — are independent to the z gap. This is because the increase in $\hat{y}_{1,t}$ are offset by the decrease in $\hat{y}_{2,t}$.

The Effect of the 2017 Federal Tax Reform

This part analyzes how would the 2017 federal tax reform affect the economy. To implement the exercise, assume the economy is initially in the steady state, in period one o_k decreases by 3.34% (about 1/3 of the steady state value) per-

[29]Recall that $\hat{l}_t \simeq \varphi\left(\hat{F}_{l,i,t} - \hat{\tau}_{p,i,t} - \hat{o}_{p,t}\right) + \hat{n}_{i,t}$.

manently, and o_p decreases by 2.24% (about 1/6 of the steady state value) temporarily.[30] The tax reform announced that most of the changes in federal personal income tax expire after 2025, this paper uses the AR(1) process to approximate the change.

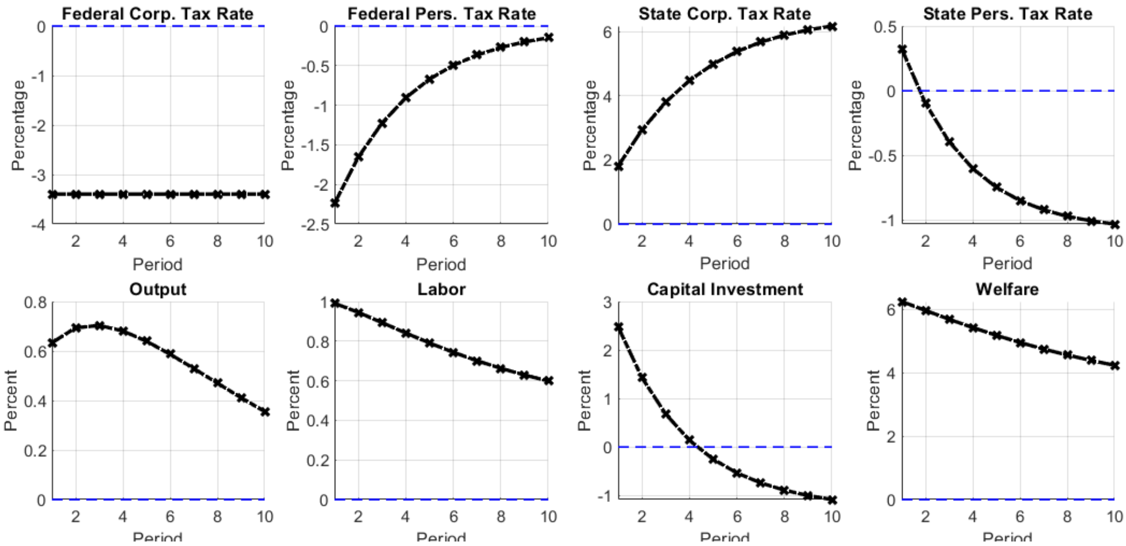

Figure 1.9: IRFs to the 2017 Federal Tax Reform

Figure 1.9 displays the impulse responses. The right two graphs of the first row display the responses of state tax rates. As can be seen, on the impact, state corporate and personal tax rates increase by 2% and 0.32% respectively. For state corporate tax rate, it increases since $\Delta\left(o_p - o_k\right) > 0$.[31] For state personal tax rate, it increases since the effect of cutting o_p (which raises τ_p) dominates the effect of cutting o_k (which lowers τ_p). Notice that the magnitudes of these initial changes are smaller than the cuts in o_k and o_p, the overall tax rates on corporate and personal income are lower than before.

[30]The federal personal income tax is progressive and the reform significantly flattens the tax rates, using the difference in *average* tax rates cannot reflect the change. Therefore, I use the difference in the *marginal* tax rate in this subsection. Using the TAXSIM program, I compute the marginal federal individual income tax rate of a single tax payer who earns the national median income of 2017 before and after the tax reform. The difference (3 percentage points) is re-scaled by the level of o_p^{ss}.

[31]Recall that the vertical component of τ_k depends upon $(1-\alpha)(o_p - o_k)$.

On the impact, the tax reform increases output by 0.64% since the lower tax rate on personal income encourages labor supply by 1%. It keeps rising in the following two periods as the lower tax rate on corporate income stimulates capital investment. The peak happens in the third period, and the value is 0.70%. After that, the response of output starts to fall. The main reason is the increase in state corporate tax rate — as the cut in o_p fades, τ_k keeps rising and the magnitude outweighs $|\Delta o_k|$.[32] Having a higher tax rate on corporate income, capital investment falls below the pre-reform level, which decreases capital and output. Moreover, the tax reform increases the social welfare by 6% on the impact, and the value slightly falls to 4% after ten periods.[33]

Table 1.5: Steady State Comparison

	τ_k^{ss}	τ_p^{ss}	Y^{ss}	L^{ss}	K^{ss}	B^{ss}
Before Reform	4.37%	5.18%	3.0266	1.8476	7.2783	0.4540
After Reform	10.65%	4.07%	3.0172	1.8540	7.1708	0.4540
Difference	6.28%	-1.11%	-0.31%	0.35%	-1.48%	0.00%

Table 1.5 compares the steady state results before and after the tax reform. Surprisingly, the tax reform decreases the long run output by 0.31%. Although labor increases by 0.35% in the long run, the over-response of state corporate tax rate reduces capital by 1.48%.

[32] The reason behind this over-response is the same as above — the capital depreciation deductibility.

[33] Since the value of V is negative, I report the response of welfare equivalence g_E such that $\chi \log(g_E) = V$.

1.5 Conclusion

This paper studies the economic effect of federal tax rate shocks under the condition that the state fiscal policies will respond to the shocks. To implement the study, I develop a two-state dynamic model with exogenous federal tax rates, benevolent state governments, freely-mobile capital, and costly-mobile labor. I show that shocks in the federal tax rates induce vertical fiscal competition because they change the marginal effect of capital or labor on the state budget constraint. Additionally, horizontal fiscal competition has the potential of altering the magnitude of a state's responses. Quantitatively, the model suggests that fiscal competition significantly influences the effect of a federal tax rate shock, in which the interaction between vertical and horizontal competition accounts for 1/5 of the change in the cumulative response of output. Moreover, the 2017 federal tax reform stimulates the short run output but decreases the long run output, because it results in an over-response in the state corporate tax rate.

To conclude, this paper emphasizes the importance of intergovernmental relationship in policy-making and shows that it is crucial in understanding the effect of a federal tax shock. My future research agenda include: i) investigating whether allowing states to have different types of public goods would affect the results of this paper; ii) studying the implication of intra-state inequality (households living in a state are heterogeneous) on state's responses to a federal shock.

Chapter 2

Optimal Policies in a Federation

2.1 Introduction

Since Ramsey (1927) and Samuelson (1954), optimal fiscal policies have received growing interest. In recent decades, there has been an increasingly large group of literature that analyzes the fiscal policies in macroeconomic models and derives important results.[1] Typically, most analyses were conducted under the setting that there is a unique fiscal authority, the central government. However, many crucial countries are using fiscal federalism — each layer of government (federal, state, and local governments) has the right to set and implement independent fiscal policies in its jurisdiction. Moreover, those governments may represent different groups' interests. Such a background makes analyzing optimal policies more challenging and intrigue. There are two main reasons: (1) governments in different layers may impose taxes on the same tax base; (2) some tax bases, especially taxes based on source principal, are mobile across

[1] For instance, Arrow and Kurz (1970), Barro (1979), Lucas and Stokey (1983), Judd (1985), Chamley (1986), Zhu (1992), Chari, Christiano, and Kehoe (1994), Chari and Kehoe (1999), Klein and Ríos-Rull (2003), Werning (2007), Bassetto (2014), Straub and Werning (2020).

regions. Therefore, governments in a federation are engaged in a fiscal competition game. The present paper wants to answer the following question: How do fiscal policies at different layers of government respond to the economic situations under the background of fiscal competition?

To proceed, I build a discrete-time dynamic model and then examine the optimal fiscal policies under commitment in a federation. Similar to Wang (1999) and Liu and Martinez-Vazquez (2015), instead of letting the players move simultaneously, I introduce a Stackelberg setting in which the federal government is the leader who internalizes the states' actions, the state governments are the followers who take the federal policies as given. Meanwhile, the states also choose policies to compete for freely mobile capital and costly mobile households to broaden the tax base and attenuate the per-household tax burden. Such a setting allows one to consider two types of fiscal competitions: vertical fiscal competition — the strategic policy setting between federal and state governments;[2] horizontal fiscal competition — the strategic policy setting among state governments.[3] Specifically, this paper is going to focus on taxes on corporate and labor income.

Before summarizing the results, it is useful to briefly introduce the setups of the model. In the modeled economy, there are two countries, the domestic country and the foreign country, and I put the attention on the domestic country. The domestic country is a federation in which the governance system is composed of the federal and two state governments. In each state, there is a continuum

[2]Papers that have similar settings include Keen (1998), Dahlby and Wilson (2003), Keen and Kotsogiannis (2002, 2004).

[3]Papers that have close settings include Wildasin (1988), Mendoza and Tesar (2005), Klein, Quadrini, and Rios-Rull (2005), Gross, Klein, and Makris (2017), and so on. However, they do not consider migration in their analysis.

of households (workers) and a perfectly competitive firm. A nationwide capitalist (capital owner) who does not live in any state supplies capital to firms in the two states and the foreign country with no cost.[4] At the beginning of each period, the infinitely-lived households make migration decisions upon observing the state variables and the assigned migration cost. After migration stops, the benevolent federal government sets the federal corporate and labor income tax rate and provides national public goods. Taking the federal taxes as given, the state governments set fiscal policies in a game similar to Cournot competition.[5] While setting policies, the state planner takes the capitalist's forward-looking condition into account, which implies that policy commitment is available. Taking each layer's fiscal policies as given, the market agents take action and then move towards the next period.

I begin with analyzing the equilibrium of the second stage of the game. At the layer of states, a crucial determinant of the optimal labor income tax rate is the marginal rate of substitution (MRS) of local public goods for household consumption. The higher is the MRS, the lower the tax rate should be, as it means the household income is relatively expensive. For the optimal corporate income tax rate, its determinants include the marginal effect of the tax rate on the interest rate (the "price effect") and the spatial allocation of capital (the "allocation effect"). The price effect affects the capital cost, the asset return, and the tightness of the Euler equation (hence the policy commitment). The allocation effect influences the tax base of corporate income tax. I find it is the ratio of these two effects that affects the optimal rate.

[4]Such a setting is similar to Judd (1985). It avoids the intra-state heterogeneity brought by migration.

[5]Each state sets policies by taking the competitor's choices as given.

As the Stackelberg followers, the states' labor and corporate income tax rates also depend upon the federal tax rates. Specifically, raising the federal labor (corporate) income tax rate increases (decreases) the optimal state corporate income tax rate. The reason is as follows. An increase in the federal labor tax rate lowers the marginal effect of capital on the state budget constraint. Because having more capital increases the marginal product of labor, the federal labor tax will transfer more resources to the federal government. Contrarily, an increase in the federal corporate tax rate decreases the marginal effect of capital. Because having more capital diminishes the marginal product of capital, the federal corporate tax will transfer fewer resources to the federal government for each unit of capital. For a similar reason, raising the federal corporate (labor) income tax rate increases (decreases) the optimal state labor income tax rate.

Now turn to the equilibrium of the first stage. I find that the federal corporate and labor income tax rates depend upon four elements: (1) the inter-state inequality; (2) the MRS of federal public goods for local public goods; (3) the effect of the tax rate on the other taxes' revenue; (4) the elasticity of the tax base to the tax rate.

The following analysis is based on the condition that the Laffer curve takes the typical shape (it holds in most of the cases during my computation). Everything else fixed, a higher degree of inter-state inequality (measured by the covariance between a state's tax base as a fraction of the total tax base and the marginal utility of local public goods) leads to a higher federal corporate or labor income tax rate. This response will redistribute more resources by the provision of national public goods. Moreover, a higher MRS of federal for local public goods leads to a lower federal corporate or labor income tax rate. Because this suggests that keeping more resources at the state level is welfare-improving as the state public goods are relatively more valuable than before. Plus, if a marginal increase in the federal corporate or labor income tax rate causes a higher loss in the other tax revenue, the optimal tax rate should be low. Lastly, being consistent with the inverse-elasticity result, the more sensitive is the tax base to the tax rate, the lower should the federal tax rate be.

Similar to Judd (1985) and Straub and Werning (2020), I am interested in the long-run tax rate on corporate income (capital) in a federation. In an interior-point steady-state where the IES of the capitalist is larger than one, the state corporate income tax rate is generally non-zero. Instead, it depends upon the federal tax rates. Because both layers of governments share the same tax base in a federation, imposing federal taxation is equivalent to reducing the states' effective total factor productivity (TFP). As a result, the gap between the effective marginal product of capital net of depreciation and the interest rate is not zero, but a function of federal tax rates. As the market agents, what matters is the overall corporate income tax rate. In this paper, I found the overall corporate tax rate is increasing in the federal labor income tax rate but has ambiguous relation to the federal corporate income tax rate. Since the capital depreciation is deductible, an increase in the federal corporate tax rate not only decreases the effective TFP but also reduces the effective depreciation rate. The two effects go against each other and the net effect depends on the value of the capital share and the depreciation rate. I show that under the widely accepted parameter values, the force that comes from depreciation deductibility outweighs the force from the changing effective TFP, so the steady-state overall corporate tax rate is a decreasing function of the federal corporate income tax rate.

By endogenizing the choice of the federal government, I found that the federal government imposes a positive and high corporate tax rate (twice as high as the labor tax rate), the state governments impose negative rates, and the overall tax rate equals zero. This zero tax rate result is robust to different degrees of inter-state inequality if the federal government has to impose the same labor tax rate on all of the states. In addition, if the federal government can impose a non-linear labor income tax system so that state-specific federal tax rate is available, the overall corporate tax rates in the two states are different. The productive state imposes a higher rate than the unproductive state because the federal government sets a higher labor tax rate due to the concern of redistribution. However, using the tax base as the corresponding weight, the weighted average tax rate in the domestic country is zero.

Assume the social welfare function is utilitarian, some interesting welfare implications could be derived from the result of this paper. First, because of migration, the total social welfare is an increasing function of inter-state inequality (approximated by the distance between state TFPs). This is because households are able to migrate to the productive state and enjoy high welfare. Second, although allowing the federal government to impose state-specific labor tax rates increases the per household welfare of the unproductive state, but it decreases the overall social welfare. Since the next migration opportunity is in the next period, the federal taxes does not affect the distribution of households. Therefore, there is an incentive to redistributing resources from the productive state to the unproductive. In the steady-state, fewer households will live in the productive state because the redistributive federal policy narrows the welfare gap. As a result, the (utilitarian) social welfare is lower than the uniform tax scenario. In this sense, if the only source of inequality comes from state-specific characteristics, a utilitarian federal government should not use a redistributive labor income tax.

By solving the global solution, I show that increasing the federal corporate tax rate is the optimal response to the scenario that the foreign country becomes more productive. Because of the vertical competition, the above tax plan lowers the state corporate tax rate and the overall tax rate on corporate income, which partly prevents capital from out-flowing. Such a result can support the tax plan proposed by president Joe Biden. Moreover, I find that a regional TFP improvement not only increases the household welfare in the home state but also increases the welfare in the neighboring state. The reason lies in two points: (1) migration gives the unproductive state residents an opportunity to move to the productive state; (2) federal government provides more nationwide public goods.

Related Literature. In addition to the literature on vertical and horizontal competition mentioned earlier, this paper relates to several groups of literature.

First of all, this is not the first paper that analyzes fiscal policies in a Stackelberg alike setting. For instance, Wang (1999) study the intergovernmental competition of commodity taxes. In this paper, the larger region is the leader and the small region is the follower. Liu and Martinez-Vazquez (2015) studies the Stackelberg equilibrium for public input competition in a static economy.

Both these two papers assume players are regions (governments) in the same layer. This paper lets the players be governments at the different layers, which allows me to consider some important issues in the fiscal competition like vertical externality. Besides, the current paper considers a dynamic framework. So the production factors are mobile not only over space but also over time.

Second, this paper also relates to the literature that analyzes the optimal capital tax rate in the long run, such as Judd (1985), Chamley (1986), Judd (1999), Le Grand and Ragot (2017), and most recently, Straub and Werning (2020). Instead of looking at just one corporate tax rate, I study the optimal tax rate at each layer of government and in different regions. Although in an aggregate point of view the optimal tax rate on capital (corporate income) is zero, the specific tax rate chosen by each layer varies by the sequence of moving. In addition, this paper also shows that both at the state level and the aggregate level, not only does the federal corporate income tax rate matter, the federal labor income tax rate also plays an important role.

Third, this paper is also built upon the literature on inter-regional migration, such as Rosen (1979), Armenta and Ortega (2010), Farhi and Werning (2014), Hsieh and Morretti (2019), and Deng (2019). The above papers consider the effects of worker mobility on redistributive policy, economic stabilization, factor miss-allocation, and regional default. Relative to them, this paper discusses the welfare implications. When inter-regional heterogeneity is the source of inequality, migration can help to achieve a higher total social welfare, and a non-linear federal labor tax system might not be desirable from the utilitarian point of view.

Outline. The rest of this paper is organized as follows: Section 2 introduces the model; Section 3 analyzes the optimal fiscal policies theoretically; Section 4 provides the quantitative analysis; Section 5 concludes.

2.2 The Model

In a specific period, the federal and state governments are engaged in a Stackelberg competition, in which the federal government is the leader and the state governments are the followers. This model is built upon Chapter 1 of my dissertation with some crucial modifications.

Timing. In each period, the timing of the model is as follows. (i) All the shocks realize, households learn their assigned migration shocks. (ii) Upon observing the state variables, households make migration decisions. (iii) The federal government sets the federal-level fiscal policies. (iv) The state governments set the state-level fiscal policies. (v) The investor allocates capital, households work, and firms produce. (vi) The investor makes investment/saving decisions and pays dividends, households consume and local governments provide public goods.

2.2.1 Household

Let $\mathbf{X}_t$ be the vector of state variables, a household initially living in state i[6] and being assigned with migration cost κ faces the following migration problem:

$$\max_{m=\{1,0\}} (1-m)V_{H,i}(\mathbf{X}_t) + m\left[V_{H,j}(\mathbf{X}_t) - \kappa\right].$$

[6]For simplicity, let state i be the home state, let state j be the neighboring state.

In the problem above, $V_{H,i}(\mathbf{X}_t)$ is the welfare of living in state i and $V_{H,j}(\mathbf{X}_t)$ is the welfare of living in state j. The solution of this binary choice problem suggests that there exists a cut-off value $\kappa_{i,t}^{j} \equiv V_{H,j}(\mathbf{X}_t) - V_{H,i}(\mathbf{X}_t)$, such that:

$$\begin{cases} m = 0 & \text{if } \kappa > \kappa_{i,t}^{j}, \\ m = 1 & \text{if } \kappa \leq \kappa_{i,t}^{j}. \end{cases}$$

Let $\Phi_{i,t}(\cdot)$ be the C.D.F of κ, the after-migration population of state i equals to:

$$n_{i,t} = \left[1 - \Phi_{i,t}\left(\kappa_{i,t}^{j}\right)\right] n_{i,t-1} + \Phi_{j,t}\left(\kappa_{j,t}^{i}\right) n_{j,t-1}. \tag{2.1}$$

After migration stops, a household deciding to live in state i solves:

$$\max_{c_{i,t}, h_{i,t}} U\left(c_{i,t}, h_{i,t}, \hat{g}_{i,t}, G_t\right)$$

subject to:

$$\left(1 - \tau_{l,i,t} - o_{l,t}\right) w_{i,t} h_{i,t} - c_{i,t} - \bar{\tau} = 0.$$

$\hat{g}_{i,t}$ represents the local public goods and it is a function of government expenditure $g_{i,t}$ and population $n_{i,t}$. G_t represents the federal public goods which is non-rivalrous and non-excludable. $\tau_{l,i,t}$ represents the state labor income tax rate. $o_{l,t}$ represents the federal labor income tax rate. The lump-sum tax $\bar{\tau}$ represents other taxes — I assume it is invariant so that the focuses are limited to taxes on production factors, one could consider $\bar{\tau}$ as tax levied by the local governments therefore it is out of state government's control.

Taking into account the necessary condition for $h_{i,t}$, the household problem must satisfy:

$$-\frac{U_{h,i,t}}{U_{c,i,t}}h_{i,t} - c_{i,t} - \bar{\tau} = 0. \tag{2.2}$$

2.2.2 Firm

Taking $w_{i,t}$ and r_t as given, the firm locates in i state faces the following problem:

$$\max_{k_{i,t},l_{i,t}} \left(1 - o_{k,t} - \tau_{k,i,t}\right)\left[z_{i,t}F\left(k_{i,t}, l_{i,t}\right) - w_{i,t}l_{i,t} - \delta k_{i,t}\right] - r_t k_{i,t}.$$

$o_{k,t}$ represents the federal-level corporate income tax rate. $\tau_{k,i,t}$ represents the state-level corporate income tax rate. Note that the taxable income equals output net of labor cost and capital depreciation. $z_{i,t}$ represents the state-specific total factor productivity (TFP), which follows:

$$\log z_{i,t} = (1 - \rho_z)\log(z_i^{s.s.}) + \rho_z \log z_{i,t-1} + \sigma_z \varepsilon_{z,i,t}.$$

The factor prices are determined by:

$$r_t = \left(1 - o_{k,t} - \tau_{k,i,t}\right)\left(z_{i,t}F_{1,i,t} - \delta\right),$$
$$w_{i,t} = z_{i,t}F_{2,i,t}.$$

The labor market clearing must satisfy $l_{i,t} = n_{i,t}h_{i,t}$.

2.2.3 Capitalist

Given the capital stock in the current period, the capitalist allocates capital among state i, j, and the foreign country w (w represents "world") with zero cost. This capital market integration assumption suggests that in the equilibrium, the following conditions must be satisfied:

$$
\begin{aligned}
k_{i,t} + k_{j,t} + k_{\mathrm{w},t} &= K_t, \\
(1 - T_{k,s,t})\left(\mathrm{MPK}_{s,t} - \delta\right) &= r_t, \\
s &= i, j, \mathrm{w}.
\end{aligned}
$$

$T_{k,s,t}$ is the overall corporate tax rate in region s. Therefore, the amount of capital that is used in state i can be written as:

$$k_{i,t} = \mathbf{K}\left(z_{i,t}, z_{j,t}, z_{\mathrm{w},t}, T_{k,i,t}, T_{k,j,t}, T_{k,f,t}, l_{i,t}, l_{j,t}, l_{\mathrm{w},t}, K_t\right), \tag{2.3}$$

and the equilibrium real interest rate is:

$$r_t = \mathbf{R}\left(z_{i,t}, z_{j,t}, z_{\mathrm{w},t}, T_{k,i,t}, T_{k,j,t}, T_{k,\mathrm{w},t}, l_{i,t}, l_{j,t}, l_{\mathrm{w},t}, K_t\right). \tag{2.4}$$

The second task of the capitalist is to maximize her lifetime utility. Formally, the problem can be written as:

$$\max_{A_{t+q+1}} \; E_t \sum_{q=0}^{\infty} \beta^q \tilde{U}(D_{t+q}),$$

subject to:

$$A_{t+q+1} + D_{t+q} = \left(1 + r_{t+q}\right) A_{t+q}. \tag{2.5}$$

A_t represents the amount of assets in the current period, such that:

$$A_t = K_t - A_{\mathrm{w},t}.^7 \tag{2.6}$$

The necessary condition for A_{t+1} is:

$$\beta E_t(1 + r_{t+1})\tilde{U}'(D_{t+1}) - \tilde{U}'(D_t) = 0. \tag{2.7}$$

2.2.4 State Government

The benevolent state government i collects corporate income tax, labor income tax, and lump-sum tax to finance the government expenditure. Taking into account the factor prices, the state budget constraint can be written as:

$$g_{i,t} = \tau_{k,i,t}(\mathrm{MPK}_{i,t} - \delta)k_{i,t} + \tau_{l,i,t}\mathrm{MPL}_{i,t}l_{i,t} + n_{i,t}\bar{\tau}.$$

In the above constraint, MPK $\equiv zF_1$, MPL $\equiv zF_2$. Using the standard transformation, the state budget constraint can be rewritten as following equation (let $\hat{w}$ be the after-tax wage):

$$z_{i,t}F(k_{i,t}, l_{i,t}) - \delta k_{i,t} + \bar{\tau}n_{i,t} = o_{k,t}(\mathrm{MPK}_{i,t} - \delta)k_{i,t} + o_{l,t}\mathrm{MPL}_{i,t}l_{i,t} + r_t k_{i,t} + \hat{w}_{i,t}l_{i,t} + g_{i,t}. \tag{2.8}$$

Let $\pi_{i,t} \equiv [T_{k,i,t}, T_{l,i,t}, g_{i,t}]$ be the vector of choice variables where $T_{k,i,t} \equiv o_{k,t} + \tau_{k,i,t}$ and $T_{l,i,t} \equiv o_{l,t} + \tau_{l,i,t}$. Let $\psi_{i,t}$ denote the multiplier associated with equation (2.2). Let $\theta_{i,t+1}$ denote the multiplier associated with equation (2.7).

[7] Since my focus is on the domestic country, I assume foreign asset is invariant overtime for the sake of simplicity.

Let $\mu_{i,t}$ denote the multiplier associated with equation (2.8). Taking $\pi_{j,t}$ and the federal policies as given, the planner of state with a after-migration population $n_{i,t}$ has the following objective function:

$$\bar{V}_i\left(n_{i,t},\mathbf{X}_t,\mathbf{o}_t,\pi_{j,t}\right)=\max_{\pi_{i,t}}\ \bar{V}_{H,i}\left(n_{i,t},\mathbf{X}_t,\mathbf{o}_t,\pi_{i,t},\pi_{j,t}\right)+\frac{\gamma}{n_{i,t}}\bar{V}_C\left(n_{i,t},\mathbf{X}_t,\mathbf{o}_t,\pi_{i,t},\pi_{j,t}\right)$$

subject to:

Equation (2.1)—(2.8).

γ is the weight associated with the capitalist's welfare and I assume it is the same in both states. Specifically,

$$\bar{V}_{H,i}\left(n_{i,t},\mathbf{X}_t,\mathbf{o}_t,\pi_{i,t},\pi_{j,t}\right)\equiv$$

$$\max_{c_{i,t},h_{i,t}} U\left(c_{i,t},h_{i,t},\hat{g}_{i,t},G_t\right)+\beta E_t\left[(1-p_{i,t+1})V_{H,i,t+1}+p_{i,t+1}V_{H,j,t+1}-\int_{-\infty}^{\kappa^j_{i,t+1}}\kappa d\Phi_{i,t+1}\right],$$

$$p_{i,t+1}\equiv\Phi_{i,t+1}\left(\kappa^j_{i,t+1}\right),$$

and

$$\bar{V}_C\left(n_{i,t},\mathbf{X}_t,\mathbf{o}_t,\pi_{i,t},\pi_{j,t}\right)\equiv$$

$$\max_{A_{t+1}}\ \tilde{U}\left((1+r_t)A_t-A_{t+1}\right)+\beta E_t V_{C,t+1},$$

$$r_t\equiv r\left(n_{i,t},\mathbf{X}_t,\mathbf{o}_t,\pi_{i,t},\pi_{j,t}\right).$$

Following Marcet and Marimon (2019), let $\Theta_{i,t}\equiv\tilde{U}'(D_t)\left[\theta_{i,t}(1+r_t)-\theta_{i,t+1}\right]$, the objective function can be rewritten as:

$$\max_{\pi_{i,t+1},A_{t+1}} \min_{\theta_{i,t+1}} U(c_{i,t}, h_{i,t}, \hat{g}_{i,t}, G_t) + \frac{\gamma}{n_{i,t}}\tilde{U}(D_t) + \Theta_{i,t}$$

$$+\beta E_t \left[(1-p_{i,t+1})V_{H,i,t+1} + p_{i,t+1}V_{H,j,t+1} - \int_{-\infty}^{\kappa_{i,t+1}^{j}} \kappa d\Phi_{i,t+1} + \frac{\gamma}{n_{i,t}}V_{C,t+1} + \Theta_{i,t+1}\right].$$

Let $\mathbf{n}_t$ be the population distribution, in the Cournot equilibrium $\pi_{s,t} = \Pi_s(\mathbf{n}_t, \mathbf{X}_t, \mathbf{o}_t)$, $\theta_{s,t+1} = T_s(\mathbf{n}_t, \mathbf{X}_t, \mathbf{o}_t)$ for $s = i, j$ and $A_{t+1} = A(\mathbf{n}_t, \mathbf{X}_t, \mathbf{o}_t)$. Define $\hat{V}_{H,s}(\mathbf{n}_t, \mathbf{X}_t, \mathbf{o}_t)$, $\hat{V}_C(\mathbf{n}_t, \mathbf{X}_t, \mathbf{o}_t)$ as the resulting indirect welfare.

Assumption: irrelevance to the future federal policies. To have a stable solution, I assume that the state governments do not take into account the influences of $\pi_{s,t}$ on $\mathbf{o}_{t+1}$ when making decisions. Such an assumption is helpful to simplify the derivation of the optimal conditions. Besides, it guarantees that the federal government is always the Stackelberg leader.

2.2.5 Federal Government

The federal government collects corporate and labor income taxes to finance government purchases G. The budget constraint reads as:

$$o_{k,t} \sum_{s=i,j} (\text{MPK}_{s,t} - \delta)k_{s,t} + o_{l,t} \sum_{s=i,j} \text{MPL}_{s,t} l_{s,t} = G_t. \tag{2.9}$$

As can be seen, transfer payment to households is not included in equation (2.9), the reason is explained in the next section.

Let $\mathbf{o}_t \equiv [o_{k,t}, o_{l,t}]$, the federal government's problem can be written as:

$$\max_{\mathbf{o}_t} \; W\left(n_{i,t}\hat{V}_{H,i}(\mathbf{n}_t, \mathbf{X}_t, \mathbf{o}_t), n_{j,t}\hat{V}_{H,j}(\mathbf{n}_t, \mathbf{X}_t, \mathbf{o}_t), \hat{V}_C(\mathbf{n}_t, \mathbf{X}_t, \mathbf{o}_t)\right)$$

subject to:

Equation (2.9).

W in the above problem represents the social welfare function. It is a function of the welfare belongs to the workers and capitalist.

2.2.6 Recursive Equilibrium

Let $\mathbf{X}_t \equiv [n_{i,t-1}, z_{i,t}, z_{j,t}, z_{\mathrm{w},t}, A_t, \theta_{i,t}, \theta_{j,t}]$. The recursive equilibrium of the model is defined as: (i) the value functions of workers in the two states and the capitalist; (ii) the distribution of population; (iii) the state and the federal policy functions. Such that:

(1) The population distribution is consistent with the value functions and equation (2.1).

(2) For $s = i, j$, taking the population distribution, the neighboring state's policies, and the federal policies as given, the state policy function and the value functions solve the state's problem.

(3) Taking the population distribution as given, the federal policy functions solve the federal government's problem.

(4) The resulting welfare values are consistent with the value functions.

2.3 Optimal State and Federal Policies

This section describes the properties of the optimal policies. From now on, let $\psi_{i,t}$ be the multiplier associated with equation (2); $\mu_{i,t}$ be the multiplier associated with equation (8). Moreover, let $U(c,h,\hat{g},G) = U\left(c - \frac{h^{1+1/\varphi}}{1+1/\varphi}\right) + I(gn^{-\eta}) + \tilde{I}(G_t)$; $F(k,l) = k^{\alpha}l^{1-\alpha}$. $W = n_{i,t}\hat{V}_{H,i,t} + n_{j,t}\hat{V}_{H,j,t} + 2\gamma\hat{V}_{C,t}$, which suggest the social welfare is utilitarian.

2.3.1 State Policies

The following session analyzes the properties of the optimal state policies.

State Corporate Tax Rate

The optimal $\tau_{k,i,t}$ must satisfy:

$$\tau_{k,i,t} = \frac{\frac{\partial r_t}{\partial \tau_{k,i,t}}}{\frac{\partial k_{i,t}}{\partial \tau_{k,i,t}}\left(\mathrm{MPK}_{i,t} - \delta\right)}\left[k_{i,t} - \frac{A_t\left[\frac{\partial \Theta_{i,t}}{\partial D_t} + \frac{\gamma}{n_{i,t}}\tilde{U}'(D_t)\right] + \tilde{U}'(D_t)\theta_{i,t-1}}{\mu_{i,t}}\right]$$
$$+\frac{(1-\alpha)(o_{l,t} - o_{k,t})\mathrm{MPK}_{i,t}}{\mathrm{MPK}_{i,t} - \delta}. \tag{2.10}$$

As can be seen, the optimal state corporate income tax rate is increasing in $o_{l,t}$ but decreasing in $o_{k,t}$. This is because the marginal effect of $k_{i,t}$ on the state budget constraint is increasing in $o_{k,t}$ but decreasing in $o_{l,t}$. Note that the marginal product of labor is increasing in k, a higher value of o_l suggests that the federal government will take more resources. Also, the marginal product of capital is decreasing in k, a higher value of o_k suggests that the federal will take fewer resources for each unit of capital. As a result, the state government will adjust its corporate tax rate to achieve the optimal level of capital. It suggests that the federal government can manipulate the overall tax rate on corporate income by adjusting its tax rates.

In addition to the vertical competition, the optimal $\tau_{k,i,t}$ also depends on its marginal effect on r_t (the price effect) and $k_{i,t}$ (the allocation effect). An increase in $\tau_{k,i,t}$ lowers r_t, and it will: (i) reduce the capital cost, (ii) decrease the value of dividend, and (iii) hurt the policy commitment made in the previous period. The higher their net effect is, the bigger is the incentive for the state to increase $\tau_{k,i,t}$. An increase in $\tau_{k,i,t}$ also decrease $k_{i,t}$, it will reduce the state's corporate tax revenue and the magnitude depends upon the per capital tax base ($\text{MPK}_{i,t} - \delta$). The higher is the net marginal product of capital, the smaller $\tau_{k,i,t}$ should be, because it means the tax base is more sensitive to the tax rate.

State Labor Tax Rate

The optimal $\tau_{l,i,t}$ must satisfy:

$$\tau_{l,i,t} = \frac{1 - \frac{\psi_{i,t}}{n_{i,t}\mu_{i,t}}}{1 + \varphi - \frac{\psi_{i,t}}{n_{i,t}\mu_{i,t}}} + \frac{\alpha\varphi}{1 + \varphi - \frac{\psi_{i,t}}{n_{i,t}\mu_{i,t}}} o_{k,t} - \frac{1 + \alpha\varphi - \frac{\psi_{i,t}}{n_{i,t}\mu_{i,t}}}{1 + \varphi - \frac{\psi_{i,t}}{n_{i,t}\mu_{i,t}}} o_{l,t}. \tag{2.11}$$

Since the labor income tax is distortionary, the marginal rate of substitution $\frac{\psi_{i,t}}{n_{i,t}\mu_{i,t}}$ is smaller than 1. Because taxing \$1 from each household not only reduces the household income but also hurts the working incentives.

As can be seen, the optimal state labor income tax rate is increasing in $o_{k,t}$ but decreasing in $o_{l,t}$. This is because the marginal effect of $l_{i,t}$ on the state budget constraint is decreasing in $o_{k,t}$ but increasing in $o_{l,t}$ (the reason is similar to the the session for $\tau_{k,i,t}$). As a result, the state government will adjust its labor tax rate to achieve the optimal level of labor. From this perspective, the federal government can manipulate the overall tax rate on labor income by adjusting its tax rates.

In addition to the vertical competition, the optimal $\tau_{l,i,t}$ also negatively relates to $\frac{\psi_{i,t}}{n_{i,t}\mu_{i,t}}$. Note that it is the ratio of shadow prices adjusted by the population, a higher value suggests that the household consumption is relatively more valuable. The higher is the ratio, the lower should $\tau_{l,i,t}$ be since the state government is willing to sacrifice more public consumption to gain one extra unit of household consumption.

Capital Stock in the next Period

Taking derivative with respect to A_{t+1}, one could obtain:

$$\frac{\partial \Theta_{i,t}}{\partial D_t}+\frac{\gamma}{n_{i,t}}\tilde{U}'(D_t) = \beta E_t\left[(1-p_{i,t+1})\frac{\partial V_{H,i}(\mathbf{X}_{t+1})}{\partial K_{t+1}}+p_{i,t+1}\frac{\partial V_{H,j}(\mathbf{X}_{t+1})}{\partial K_{t+1}}+\frac{\gamma}{n_{i,t}}\frac{\partial V_{C,t+1}}{\partial A_{t+1}}\right].$$

$$+\beta E_t\left[\left(1+r_{t+1}+\frac{\partial r_{t+1}}{\partial K_{t+1}}A_{t+1}\right)\frac{\partial \Theta_{i,t+1}}{\partial D_{t+1}}\right] \tag{2.12}$$

Equation (2.12) describes the evolution of $\theta_{i,t}$ — the co-state of the recursive problem. A marginal increase in A_{t+1} reduces the amount of D_t and cause the saving incentive to change accordingly. In the optimum, this marginal loss must equal the marginal impact of A_{t+1} on the future welfare. Using the Envelope theorem, one can show that:

$$\frac{\partial V_{H,i,t+1}}{\partial A_{t+1}} \equiv \frac{\partial k_{i,t+1}}{\partial K_{t+1}}\frac{\partial g_{i,t+1}}{\partial k_{i,t+1}}\mu_{i,t+1}-\frac{\partial r_{t+1}}{\partial K_{t+1}}k_{i,t+1}\mu_{i,t+1}+\frac{\partial n_{i,t+1}}{\partial K_{t+1}}\frac{\partial \hat{V}_{H,i,t+1}}{\partial n_{i,t+1}},$$

$$\frac{\partial V_{C,t+1}}{\partial A_{t+1}} \equiv (1+r_{t+1})\tilde{U}'(D_{t+1}) + \frac{\partial r_{t+1}}{\partial K_{t+1}} A_{t+1}\tilde{U}'(D_{t+1}).$$

Combining equation (2.12) with equation (2.7) and (2.10), in the interior-point steady-state that $\beta(1+r)=1$, one can obtain that:

$$\frac{\partial g_i}{\partial k_i} \equiv [\tau_{k,i} + (1-\alpha)(o_k - o_l)]\, \text{MPK}_i - \delta\tau_{k,i} = 0,$$

$$\Rightarrow \tau_{k,i} = \frac{(1-\alpha)(o_l - o_k)\text{MPK}_i}{\text{MPK}_i - \delta}.$$

According to the necessary condition for $\tau_{k,i,t}$, this result suggests that the horizontal competition component of state corporate tax rate is zero and $\tau_{k,i}$ equals to its vertical component. Intuitively, when the two states have stopped changing policies and the economy has been in a steady situation, the role of capital competition disappears as there is no inter-regional flow of capital, therefore τ_k only depends upon the vertical competition.

This finding could be considered as a modified Charmley-Judd result. In the case that $o_k = o_l$, the state corporate tax rate equals zero. However, when $o_k \neq o_l$, the state corporate tax equals a non-zero number. In terms of the overall tax rate, one can show that:

$$T_{k,i} \equiv o_k + \tau_{k,i} = \frac{(1-\alpha)o_l + \left(\alpha - \frac{\delta}{\text{MPK}_i}\right) o_k}{1 - \frac{\delta}{\text{MPK}_i}}.$$

It suggests that the steady-state overall tax rate on corporate income (capital) is increasing in o_l, but has an ambiguous relationship to o_k.

The reason lies in the setting that capital depreciation is deductible from the corporate income. Using the Cobb-Douglas production function, one can rewrite equation (2.8) as:

$$(1-\alpha o_{k,t}-(1-\alpha)o_{l,t})z_{i,t}k_{i,t}^{\alpha}l_{i,t}^{1-\alpha}-\delta(1-o_{k,t})k_{i,t}+\bar{\tau}n_{i,t}=g_{i,t}+r_tk_{i,t}+h_{i,t}^{1+\frac{1}{\varphi}}n_{i,t}.$$

Notice that an increase in o_k not only declines the effective TFP (the coefficient associated with $z_{i,t}$ is smaller) but also lowers the effective depreciation rate (the coefficient associated with δ is smaller) because of the deductibility. As a result, it has an ambiguous effect on the net marginal product of capital. The zero corporate tax result says that the gap between the effective net marginal product of capital and the interest should be zero. In case that the net marginal product of capital increases with o_k, an raise in o_k will cause a gap that requires a bigger response of $\tau_{k,i}$, thus the optimal $T_{k,i}$ becomes lower. Otherwise, the $T_{k,i}$ is increasing in o_k.

In the interior-point steady-state,

$$r=\frac{1}{\beta}-1=(1-T_{k,i})(\mathrm{MPK}_i-\delta)\Rightarrow \mathrm{MPK}_i=\frac{\frac{1}{\beta}-1}{1-T_{k,i}}+\delta.$$

Plug into the equation for $T_{k,i}$,

$$T_{k,i}=\frac{(1-\alpha)o_l+\left(\alpha-\frac{\delta}{\frac{\frac{1}{\beta}-1}{1-T_{k,i}}+\delta}\right)o_k}{1-\frac{\delta}{\frac{\frac{1}{\beta}-1}{1-T_{k,i}}+\delta}},$$

$$\Rightarrow T_{k,i}=\frac{\left(\frac{1}{\beta}-1\right)f}{\frac{1}{\beta}-1+\delta(1-\alpha)(o_l-o_k)}+\frac{\delta(1-\alpha)(o_l-o_k)}{\frac{1}{\beta}-1+\delta(1-\alpha)(o_l-o_k)},\ f\equiv\alpha o_k+(1-\alpha)o_l.$$

If $\delta=0$, then $T_{k,i}$ equals the weighted-average federal tax rate f. In general cases that $\delta \neq 0$, the first term, the effect comes from the falling effctive TFP, is increasing in o_k; and the second term, the effect comes from the deductibility, is decreasing in o_k.

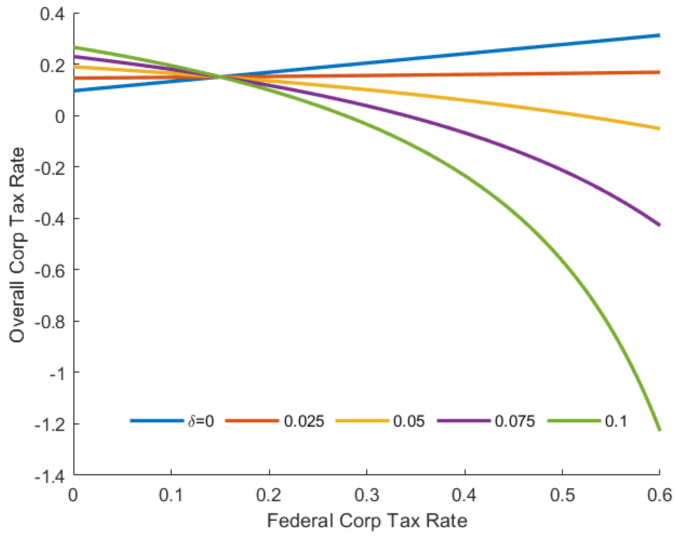

Figure 2.1: Overall Tax Rate on Corporate Income

It is useful to show the relationship between o_k and T_k graphically. Let the time frequency be a year, I set $\beta = 0.96$, $o_l = 0.15$ (it does not affect the relation qualitatively), $\alpha = 0.36$. The above figure plots $T_{k,i}$ against o_k when δ takes values from 0 to 0.1. As shown in figure 2.1, $T_{k,i}$ is decreasing in o_k when δ is sufficiently big (under the current setting, the critical value is around 0.03). $T_{k,i}$ is increasing in o_k when δ is smaller than the critical value. Given the time frequency, $T_{k,i}$ is a decreasing function of o_k when one chooses widely accepted values of δ.

2.3.2 Federal Policies

The federal government's budget constraint can be written as:

$$f_t \sum_{s=i,j} y_{s,t} - \delta o_{k,t} \sum_{s=i,j} k_{s,t} \equiv f_t Y_{d,t} - \delta o_{k,t} K_{d,t} = G_t.$$

Where $f_t = \alpha o_{k,t} + (1-\alpha) o_{l,t}$, $Y_{d,t} \equiv y_{i,t} + y_{j,t}$, $K_{d,t} \equiv k_{i,t} + k_{j,t}$.

Federal Public Expenditure

Notice that $\frac{\hat{V}_{H,s,t}}{\partial G_t} = \tilde{I}'(G_t)$, let Λ_t be the multiplier associated with the federal budget constraint, the necessary condition for G_t reads as:

$$\Lambda_t = \bar{n}\tilde{I}'(G_t). \tag{2.13}$$

Equation (2.13) suggests that the shadow price of the federal expenditure is proportional to the marginal utility of G_t and the coefficient is the total population in the domestic country.

Federal Corporate Income Tax Rate

Using the Envelope theorem,

$$\frac{\partial \hat{V}_{H,s,t}}{\partial o_{k,t}} = \mu_{s,t}\left(-\alpha y_{s,t} + \delta k_{s,t}\right).$$

$$\frac{\partial \hat{V}_{C,t}}{\partial o_{k,t}} = 0.$$ [8]

[8]Note that the state government's choice variable is the overall corporate tax rate $T_{k,s,t}$, $o_{k,t}$ does not exist in the capitalist's problem.

Taking the necessary condition for $o_{k,t}$, one could obtain:

$$\Lambda_t \left[\alpha Y_{d,t} - \delta K_{d,t} + f_t \frac{\partial Y_{d,t}}{\partial o_{k,t}} - \delta o_{k,t} \frac{\partial K_{d,t}}{\partial o_{k,t}}\right] = \sum_{s=i,j} n_{s,t}\mu_{s,t}\left(\alpha y_{s,t} - \delta k_{s,t}\right).$$

Dividing Λ_t on both sides and make some rearrangements,

$$f_t \frac{\partial Y_{d,t}}{\partial o_{k,t}} - o_{k,t}\delta \frac{\partial K_{d,t}}{\partial o_{k,t}} = \sum_{s=i,j}\left(\alpha y_{s,t} - \delta k_{s,t}\right)\left(\frac{n_{s,t}\mu_{s,t}}{\Lambda_t} - 1\right). \tag{2.14}$$

Assume the the federal tax base is decreasing in o_k, the left-hand-side of equation (2.14) is negative. It can be shown that:

$$\sum_{s=i,j}\left(\alpha y_{s,t} - \delta k_{s,t}\right)\left(\frac{n_{s,t}\mu_{s,t}}{\Lambda_t} - 1\right) < 0 \Rightarrow \sum_{s=i,j} x_{k,s,t} n_{s,t}\mu_{s,t} \leq \Lambda_t.$$

$x_{k,s,t}$ is the share of corporate income tax base that belongs to state s: $x_{k,s,t} \equiv \frac{\alpha y_{s,t} - \delta k_{s,t}}{\alpha Y_{d,t} - \delta K_{d,t}}$. Therefore, the marginal utility of federal public goods is higher than that of local public goods in a weighted-average sense. In other words, the federal budget constraint is tighter than the states' constraint.

By simple rearrangements, equation (2.14) becomes:

$$o_{k,t} = \frac{\frac{1}{\bar{n}}E\left(\frac{n_{s,t}\mu_{s,t}}{\Lambda_t} - 1\right) + \mathrm{COV}\left(x_{k,s,t}, \frac{n_{s,t}\mu_{s,t}}{\Lambda_t}\right) - \frac{1}{\bar{n}}\frac{o_{l,t}(1-\alpha)\frac{\partial Y_{d,t}}{\partial o_{k,t}}}{\alpha Y_{d,t} - \delta K_{d,t}}}{\frac{1}{\bar{n}}\varepsilon^{o_k}_{K,t}},$$

$$\varepsilon^{o_k}_{k,t} \equiv \frac{\partial\left(\alpha Y_{d,t} - \delta K_{d,t}\right)}{\partial o_{k,t}} \frac{1}{\alpha Y_{d,t} - \delta K_{d,t}}.$$

The above formula implies that the optimal federal corporate tax rate depends on four determinants: the inter-state inequality $\mathrm{COV}\left(x_{k,s,t}, \frac{n_{s,t}\mu_{s,t}}{\Lambda_t}\right)$; the average marginal rate of substitution $E\left(\frac{n_{s,t}\mu_{s,t}}{\Lambda_t}\right)$; the effect of o_k on the labor income tax; and the tax base sensitivity $\varepsilon_{k,t}^{o_k}$.

The intuitions of the above formula are as below (assume the case that $\varepsilon_{K,t}^{o_k} < 0$). Everything else fixed: (1) a decrease in the co-variance — note that it is negative — means the degree of inter-state inequality is higher, which suggests that the $o_{k,t}$ should increase to enhance the redistribution; (2) an increase in $E\left(\frac{n_{s,t}\mu_t}{\Lambda_t}\right)$ suggests that the marginal rate of substitution of G for g is higher, the optimal $o_{k,t}$ should be decrease since the federal government is willing to sacrifice more federal public goods to obtain one more unit of state public good; (3) when rising $o_{k,t}$ induces a higher loss in the federal labor income tax (the third term in the numerator becomes more negative), the optimal $o_{k,t}$ should decrease, which represents a trade-off between the two types of taxes; (4) a decrease in $\varepsilon_{K,t}^{o_k}$ suggests that the corporate tax base is more sensitive to $o_{k,t}$, hence the optimal $o_{k,t}$ should decrease to reduce the inefficiency.

Federal Labor Income Tax Rate

Using the Envelope theorem,

$$\frac{\partial \hat{V}_{H,s,t}}{\partial o_{l,t}} = -\mu_{s,t}(1-\alpha)y_{s,t},$$

$$\frac{\partial \hat{V}_{C,t}}{\partial o_{l,t}} = 0.$$

The necessary condition for $o_{l,t}$ suggests that the optimal federal labor income tax rate must satisfy:

$$\Lambda_t\left[(1-\alpha)Y_{d,t} + f_t\frac{\partial Y_{d,t}}{\partial o_{l,t}} - o_{k,t}\delta\frac{\partial K_{d,t}}{\partial o_{l,t}}\right] = (1-\alpha)\sum_{s=i,j} n_{s,t}\mu_{s,t}y_{s,t},$$

$$\Rightarrow f_t \frac{\partial Y_{d,t}}{\partial o_{l,t}} - o_{k,t}\delta \frac{\partial K_{d,t}}{\partial o_{l,t}} = (1-\alpha) \sum_{s=i,j} y_{s,t} \left(\frac{n_{s,t}\mu_{s,t}}{\Lambda_t} - 1 \right). \tag{2.15}$$

Similarly to the session above, equation (2.15) implies that if $o_{l,t}$ has negative effect on the federal tax base, one could obtain:

$$\sum_{\text{s=i,j}} n_{s,t}\mu_{s,t}x_{y,s,t} < \Lambda_t.$$

Moreover, equation (2.15) implies that:

$$o_{l,t} = \frac{\frac{1}{\bar{n}} E\left(\frac{n_{s,t}\mu_{s,t}}{\Lambda_t} - 1\right) + \text{COV}\left(x_{y,s,t}, \frac{n_{s,t}\mu_{s,t}}{\Lambda_t}\right) - \frac{1}{\bar{n}} \frac{\alpha o_{k,t}\frac{\partial Y_{d,t}}{\partial o_{l,t}} - o_{k,t}\delta\frac{\partial K_{d,t}}{\partial o_{l,t}}}{(1-\alpha)Y_{d,t}}}{\frac{1}{\bar{n}}\varepsilon_{l,t}^{o_l}},$$

$$\varepsilon_{l,t}^{o_l} \equiv \frac{\partial(1-\alpha)Y_{d,t}}{\partial o_{l,t}} \frac{1}{(1-\alpha)Y_{d,t}}.$$

The intuitions of the above formula are the same as in the previous part.

Transfer Payment

One may realize that transfer payment to households does not enter the federal budget constraint. The reason lies in the following proposition.

Proposition: zero transfer payment. *If the distribution of population is sufficiently close to the distribution of output and raising federal taxes shrinks the tax base, then the optimal transfer payment is zero.*

Proof: By using the envelope theorem, the marginal effect of transfer payment on $\hat{V}_{H,s,t}$ equals to $\psi_{s,t}$. Therefore the necessary condition for the transfer payment is:

$$\Lambda_t \geq \sum_{s=1,2} \frac{n_{s,t}}{\bar{n}} \psi_{s,t} \equiv \sum_{s=1,2} x_{n,s,t}\psi_{s,t}$$

$$\text{Transfer Payment} = 0 \text{ if it is strictly unequal.}$$

Based on the result above, when the federal tax base is negatively related to the tax rates,

$$\Lambda_t > \sum_{s=i,j} x_{y,s,t} n_{s,t} \mu_{s,t} > \sum_{s=i,j} x_{y,s,t} \psi_{s,t}.$$

The second part of the inequality comes from the result that $n_{s,t}\mu_{s,t} > \psi_{s,t}$ since the state tax system is distortionary. As long as $x_{n,s,t}$ and $x_{y,s,t}$ are close enough, the inequality will hold and thus the optimal federal transfer payment is zero. ■

Intuitively, collecting federal revenue and then transferring it to households is inefficient due to the associated distortion. A better way to increase household income is to lower the federal labor tax rates. Using the similar reasoning, the optimal state transfer payment is zero when $n_{s,t}\mu_{s,t} > \psi_{s,t}$.

2.4 Quantitative Analysis

In this section, I calibrate the model to match the inter-state migration data and public finance data. The benchmark is the case that the two states are identical in the steady-state. To ensure a interior-point steady-state, I assume $\tilde{U}(D) = \frac{D^{1-\sigma}-1}{1-\sigma}$ and $\sigma < 1$.[9] For the other functional forms, I set $\Phi_{i,t}(\kappa) = \left[1 + exp\left(-\frac{\kappa - \left(\mu_\kappa - \beta_\kappa \log z_{i,t}/z_i^{s.s}\right)}{\sigma_\kappa}\right)\right]^{-1}$,[10] $U(c,h) = \log\left(c - \frac{h^{1+1/\varphi}}{1+1/\varphi}\right)$, $I\left(gn^{-\eta}\right) = \chi_1 \log(gn^{-\eta})$, $\tilde{I}(G) = \chi_2 \log(G)$.

The unit of time is set to a year, which captures the fact that the federal

[9]See Straub and Werning (2020) for the related intuition. In this paper, I find that there is no interior-point steady-state when $\sigma = 1$. The capital stock goes to the lower bound (if any) when $\sigma > 1$.

[10]This suggests that κ follows a Logistic distribution with mean $\mu_\kappa - \beta_\kappa \log(z)$ and standard deviation σ_z. Having the mean negatively correlated to z implies that the correlation between worker outflow and inflow is positive at the state level, which is consistent with the data.

Table 2.1: Parameter Values

Parameter	Meaning	Value
β	subjective discount factor	0.96
α	capital share	0.36
δ	depreciation rate	0.10
φ	labor elasticity	0.5
χ_1	preference of local public goods	0.3650
χ_2	preference of federal public goods	0.4010
σ	inverse of IES for the capitalist	0.5
γ	weight associated with the capitalist's welfare	0
A_{w}	asset owned by foreign country	8.3216
$T_{k,\text{w}}$	foreign corporate tax rate	0.19
$T_{l,\text{w}}$	foreign labor tax rate	0.20
μ_κ	mean of migration shocks	1.7380
σ_κ	standard deviation of migration shocks	0.5
β_κ	relation between mean migration cost and TFP	0.883
η	degree of congestion for local public goods	0.35
ρ_z	persistence of TFP shocks	0.87

and state governments set policies every fiscal year. In line with other macroeconomics literature, the following parameter values are assigned: $\beta = 0.96$, $\delta = 0.1$[11], $\varphi = 0.5$, $\alpha = 0.36$, and $\sigma = 0.5$ (IES=2). $\rho_z = 0.87$ comes from the FRED data set. For the foreign country, I set $T_{k,\text{w}} = 0.19$, $T_{l,\text{w}} = 0.20$ and $n_{\text{w}} = 2$. These settings are close to the situation of the European Union.

The set $[\chi_1, \bar{\tau}, A_{\text{w}}]$ is calibrated to match moments under the scenario that (i) n_i and n_j are constant; (ii) $r = \frac{1}{\beta} - 1$; (iii) $\frac{y_i/n_i}{y_j/n_j} = 1.26$; (iv) $\frac{G}{Y_d} = 0.1$; (v) $\frac{\text{State Labor Income Tax}}{\text{State Corporate Income Tax}} = 6.40$. Condition (i) and (ii) suggest the economy is invariant over time. However, it is not an interior-point steady-state situation because, instead of endogenizing o_k and o_l, condition (iii) and (iv) have fixed their values. I found that the interior-point steady-state implies negative state corporate

[11]Notice that this suggests the overall corporate tax rate is decreasing in o_k as is discussed above.

tax rates that are rare in the real-life, therefor I pick the above situation as the calibration benchmark as it is close to the U.S. data from 1986 to 2016.[12] The moments I pick include: $\frac{g_i+g_j}{y_i+y_j} = 0.10$; $\frac{2\bar{\tau}}{g_i+g_j} = 0.70$; in the net asset position, the amount of U.S. capital belongs to foreign countries equals 40% of output. χ_2 is calibrated to match the steady-state $\frac{G}{Y_d} = 0.1$. The rest of model parameters are estimated by simulation.

2.4.1 Steady-State Performances

The baseline steady-state results are listed in table 2.2. In this benchmark case, the two states are identical so that $V_{H,i} = V_{H,j}$, $n_i = n_j$, and $z_i = z_j$.

The first result worthwhile mentioning is that $o_k = 0.2180$ and $\tau_k = -0.2180$, which means the overall tax rate on corporate income (capital) is 0. The intuition is straightforward: since the two states are identical, the federal planner's objective function looks as if there is a unique labor supplier and a capitalist. If the federal government is going to choose tax rates endogenously, the classic Chamley-Judd result must hold. Even though the overall rates are zero, the tax rates in different layers are different. As the Stackelberg leader, the federal government takes the states' reactions into account and then sets a high o_k, while the Stackelberg followers — state governments — set negative $\tau_{k,s}$.

Such a result implies that the federal corporate income tax rate is almost twice as high as the federal labor income tax rate. In 2020, the U.S. average corporate and labor tax rates are 0.0667 and 0.1008. In this sense, president Joe Biden's tax plan that increases the corporate tax rate and lowers the individual

[12] For condition (iii), I assume state i is the state whose output per capita is ranked 13 (25 percentile) and state j is the one whose output per capita is ranke 37 (75 percentile).

Table 2.2: Steady-State Performance: benchmark

o_k	o_l	G	τ_k	τ_l	g
0.2180	0.1178	0.3098	-0.2180	0.0814	0.1526
y	k	h	c	D	θ
1.5725	3.9960	0.9306	0.6989	0.2806	10.1257
V_H	V_C	W			
-1.6330	-0.9406	-3.2660			

tax rate is welfare-improving in the steady-state. The figure below depicts W as function of o_k and o_l in the steady-state.

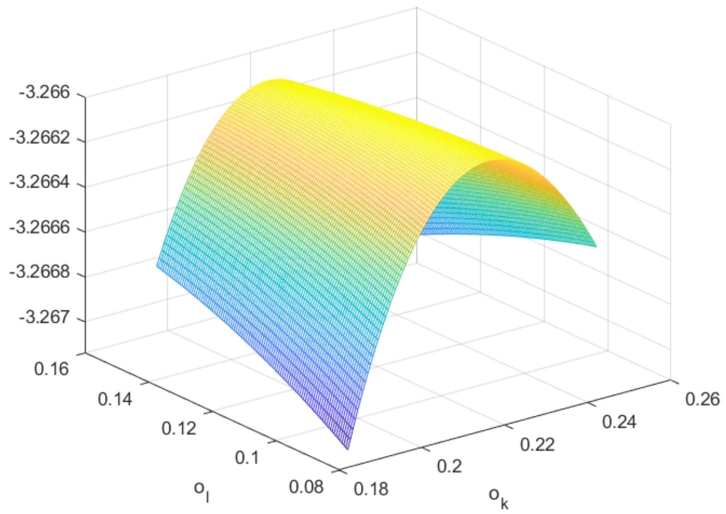

Figure 2.2: Steady-State Social Welfare against Federal Taxes

Figure 2.2 shows that the steady-state social welfare is a non-monotonic function of o_k. When o_k is small, raising o_k increases W rapidly since the marginal utility of G is high. As o_k becomes bigger and bigger, the social welfare gradually declines as the loss of economic output (since it increases τ_l and hence lowers h and l) dominates its benefit. Conditional on a low o_k, the social welfare is increasing in o_l because it brings more federal publics. However, conditional on a high o_k, the social welfare is decreasing in o_l. Because there has been plenty of federal public goods, continuing rising o_l increases the overall tax rates on corporate and labor income and hence hurts the economy. The marginal benefit and cost are balanced at the point (0.2180, 0.1178) where the resulting welfare is at the peak.

Change Capitalist's Weight

The above table is based on the benchmark case that $\gamma = 0$. It would be interesting to explore the case that $\gamma \neq 0$.

Table 2.3: Steady-state Performance: varying γ

γ	o_k	o_l	G	τ_k	τ_l	g	V_H	V_C
0	0.2180	0.1178	0.3098	-0.2180	0.0814	0.1526	-1.6330	-0.9406
0.0001	0.2199	0.1175	0.3097	-0.2233	0.0822	0.1526	-1.6330	-0.9396
0.0010	0.2364	0.1146	0.3092	-0.2717	0.0890	0.1526	-1.6331	-0.9311
0.0100	0.3850	0.0937	0.3016	-0.8356	0.1478	0.1526	-1.6443	-0.8486

Table 2.3 shows the results when the weight associated with V_C varies from 0.0001 to 0.01. As γ goes up, the transfer from workers to the capitalist gradually increases. The overall tax rate on corporate income decreases from -0.0034 in row two to -0.4506 in row 4. Another evidence is that the value of federal public expenditure declines from 0.3097 to 0.3016. Moreover, the social welfare of households (workers) declines while the welfare of the capitalist increases.

Vary Inter-State Inequality

In the next exercise, I explore the effect of inter-state inequality on the steady-state results. The inequality is defined as the gap between z_i and z_j while $z_i + z_j = 2$. In the table below, I list the results when the inter-state TFP gap enlarges from 0.02 to 0.12. [13]

[13] The output-per-capita ratio of the 25th to 75th state requires a gap of around 0.1.

Table 2.4: Steady-state Performance: inter-state inequality

$z_i - z_j$	o_k	o_l	$\tau_{k,i}$ & $\tau_{k,j}$	$\tau_{l,i}$ & $\tau_{l,j}$	$V_{H,i}$ & $V_{H,j}$	n_i	W
0.02	0.2180	0.1178	-0.2180 & -0.2180	0.8037 & 0.0790	-1.6101 & -1.6538	1.0842	-3.2602
0.04	0.2180	0.1178	-0.2180 & -0.2180	0.0860 & 0.0765	-1.5853 & -1.6723	1.1666	-3.2431
0.06	0.2180	0.1178	-0.2180 & -0.2180	0.0882 & 0.0740	-1.5588 & -1.6886	1.2457	-3.2154
0.12	0.2179	0.1178	-0.2179 & -0.2179	0.0943 & 0.0658	-1.4714 & -1.7239	1.4523	-3.0811

Table 2.4 indicates that the inter-state inequality has little effect on the federal tax rates. Intuitively, rising inter-state inequality decreases the covariance between a state's tax base share and the G for g MRS, which increases the optimal federal tax rate. Besides, it also increases the feasible resources since factor mobility suggests that more factors can work in the productive state, which decreases the optimal federal tax rate. These two effects offset each other in the steady-state. While the vertical component of τ_k changes little, in the horizontal component, the price effect and allocation effect are also canceled out, so the state corporate tax rate is invariant. As a result, the overall tax rates on corporate remain 0 when the degree of inequality changes. Since more household moves to the productive state, in the state i, an increase in $\tau_{l,i}$ can raise more tax revenue due to the bigger amount of taxpayers, thus the optimal $\tau_{l,i}$ increases with $z_i - z_j$. In contrast, the labor tax rate in the unproductive state decreases as $z_i - z_j$ goes up. Lastly, the table shows that the value of W is increasing in the TFP gap. Because of migration, more and more households migrate from state j to i and enjoy the high welfare. Since the social welfare function is utilitarian, the total social welfare improves with the household redistribution.

Allow State-Specific Labor Tax Rate

The assumption that the same federal tax rate applies to both states limits the redistribution effect of the federal policies. In this part, I allow the federal labor income tax to be nonlinear so that $o_{l,i}$ and $o_{l,j}$ can be different. With state-specific o_l, the condition becomes:

$$o_{l,s,t} = \frac{\left(1 - \frac{n_{s,t}\mu_{s,t}}{\Lambda_t}\right) + \frac{\alpha o_{k,t}\frac{\partial y_{s,t}}{\partial o_{l,s,t}} + f_{s',t}\frac{\partial y_{s',t}}{\partial o_{l,s,t}} - \delta o_{k,t}\frac{\partial K_{d,t}}{\partial o_{l,s,t}}}{(1-\alpha)y_{s,t}}}{\left|\varepsilon^{o_l}_{l,s,t}\right|}.$$

Now, the value of o_l depends on the state's MRS of G for g, the effect on the rest of federal tax revenue, and the tax base sensitivity. Everything else fixed, a productive state is going to have a higher o_l since the MRS is smaller than the unproductive state.

Table 2.5: Inter-state Inequality and State-Specific o_l

$z_i - z_j$	o_k	$o_{l,i}$ & $o_{l,j}$	$\tau_{k,i}$ & $\tau_{k,j}$	$V_{H,i}$ & $V_{H,j}$	n_i	W
0.02	0.2180	0.1252 & 0.1097	-0.1993 & -0.2393	-1.6229 & -1.6409	1.0348	-3.2631
0.04	0.2180	0.1319 & 0.1007	-0.1828 & -0.2635	-1.6106 & -1.6465	1.0693	-3.2545
0.06	0.2180	0.1378 & 0.0909	-0.1684 & -0.2907	-1.5961 & -1.6498	1.1035	-3.2403
0.12	0.2179	0.1518 & 0.0566	-0.1356 & -0.3950	-1.5409 & -1.6469	1.2020	-3.1663

Table 2.5 shows that the value of o_k stays the same as $z_i - z_j$ rises. However, $o_{l,i}$ and $o_{l,j}$ becomes different as the two states are now heterogeneous. As can be seen, the tax rate imposed on the productive state is higher than the counterpart imposed on the unproductive state, and the gap is widening from row 2 to row 5. Such a schedule suggests that the labor income tax system is progressive and it redistributes resources from state i to state j by providing federal public goods. As a result, the gap between $V_{H,i}$ and $V_{H,j}$ is smaller than the result under a linear tax system. Interestingly, allowing non-linear federal labor income tax does not

increase social welfare. Instead, the last column in table 2.5 is smaller than the corresponding column in table 2.4. Because migration takes place before the federal government setting policies, $o_{l,i}$ and $o_{l,j}$ have no effect on the population distribution. Taking n_i and n_j as given, there exists an incentive to boost $V_{H,j}$ via a lower $o_{l,j}$. In the end, relative to table 2.4, fewer households will live in the productive state in table 2.5, which lowers the social welfare.

What happens to the zero tax rate result? Table 2.5 shows that neither $T_{k,i} \equiv o_k + \tau_{k,i}$ or $T_{k,j} \equiv o_k + \tau_{k,j}$ is zero. This is because $o_{l,i}$ and $o_{l,j}$ also enters the vertical component of τ_k and the redistribution incentive makes them different. However, I found that if one uses $\alpha y_s - \delta k_i$ — the state's corporate income tax base — as the weight, then $\sum_s x_{k,s}T_{k,s} = 0$.

Table 2.6: Zero Tax Rate Result: Revisit

$z_i - z_j$	$\alpha y_i - \delta k_i$	$\alpha y_j - \delta k_j$	$T_{k,i}$	$T_{k,j}$	$x_{k,i}T_{k,i} + x_{k,j}T_{k,j}$
0.02	0.1773	0.1560	0.0187	-0.0213	0.0000
0.04	0.1883	0.1478	0.0352	-0.0454	0.0000
0.06	0.1995	0.1360	0.0496	-0.0727	0.0000
0.12	0.2341	0.1088	0.0823	-0.1771	0.0000

2.4.2 Global Solution and Policy Functions

In this section, I study the model quantitatively in a global sense. The computation algorithm is in the appendix. Particularly, my plan is to understand how would the state variables, like foreign TFP, affect the federal and state policies as well as the economic results.

The Effect of Foreign Country's TFP

In this exercise, I vary the foreign country's TFP from 0.97 to 1.03 and analyze its effect on fiscal policies and welfare.

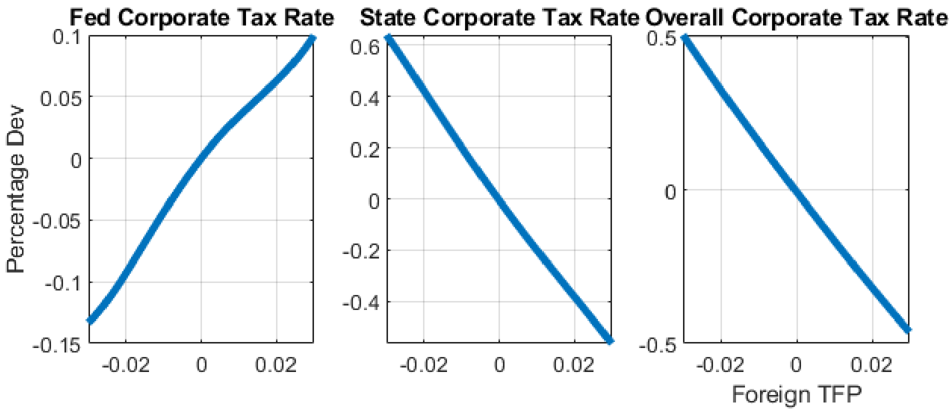

Figure 2.3: The Effect of Foreign TFP: Corporate Income Tax

To have a comparable result, the numbers on the vertical axis are the percentage deviations from the interior-point steady-state. Figure 2.3 shows that facing a rising foreign TFP, the federal corporate income tax rate slightly increases from 21.66% to 21.90% (the step is about 24 basis points). The intensive competition and federal government's move decline the state corporate tax rate by about 1.2%, making the overall tax rate on corporate income a decreasing function of the foreign TFP. The above figure indicates that the current Biden Tax Plan that increases the federal corporate income tax rate and the top marginal tax rate of the federal individual income tax is an optimal response to the situation that the foreign countries are more productive.

The numbers on the vertical axis in figure 2.4 are the percent deviations from the interior-point steady-state. Figure 2.4 shows that increasing the foreign TFP by 0.06 will decrease the domestic output by about 2 percent relative to the steady-state value. Household welfare also decreases because of capital outflow

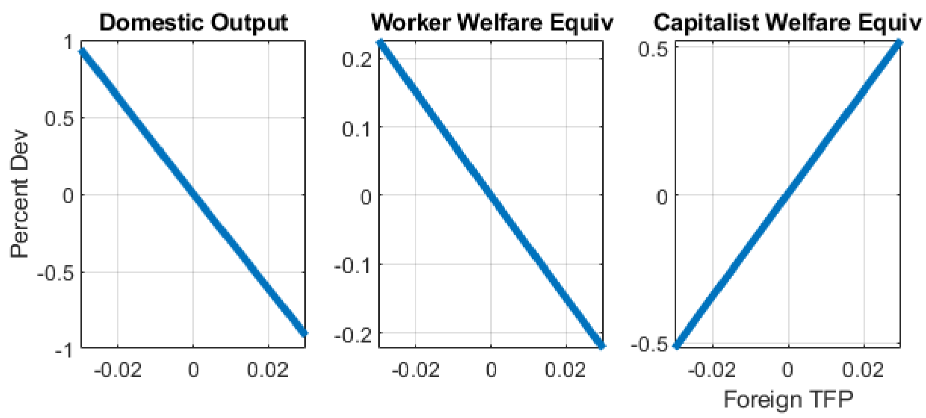

Figure 2.4: The Effect of Foreign TFP: Output and Welfare

and the resulting low marginal product of labor. The welfare equivalence decreases by 0.44 percent compared to the steady-state value. Since the capital market integration allows the capitalist to invest in the foreign country, the welfare equivalence of V_C rises from -0.5% below the steady-state value to +0.5% above.[14]

Transitional Regional TFP Innovation

This exercise studies the effect of a transitional regional TFP innovation on fiscal policies and the economic consequences. The size of the innovation ranges from -0.03 to 0.03.

Looking at the policy functions at the federal level, an increase in the home state's TFP increases the corporate income tax rate but decreases the labor income tax rate. The magnitude is relatively small (the highest deviation from steady-state is ten basis points). Intuitively, with the increase in Z_i, the inter-state inequality enlarges, which suggests the tax rates should increase. However, the higher regional TFP also attracts more foreign capital and relaxes the

[14]For the households, I define the welfare equivalence as c_{eq} such that $\log(c_{eq}) = V_H$. For the capitalist, I define the welfare equivalence as D_{eq} such that $\frac{D_{eq}^{1-\sigma}-1}{1-\sigma} = V_C$.

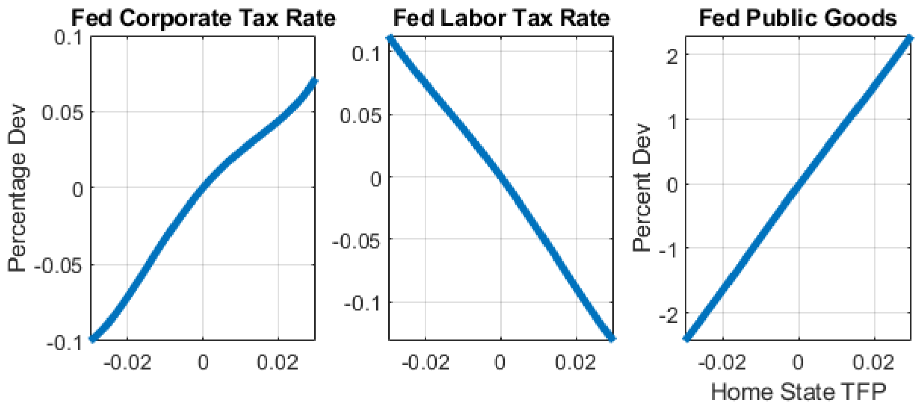

Figure 2.5: The Effect of Z_i: Federal Policies

domestic resource constraint, so the tax rates should be lower. The cross-tax effect does not change much since both the loss of the other tax revenue and the tax base increase. Lastly, the tax base elasticity of the federal corporate tax rate slightly decreases (note that increasing o_k can reduce τ_k and prevent capital from out-flowing), the elasticity of the federal labor tax rate slightly increases (note that increasing o_l increases the overall tax rates on both corporate and labor income). As a result, people could see the path in the first two graphs. Although the tax rates do not change much, the federal public goods provision increases due to the broader tax base.

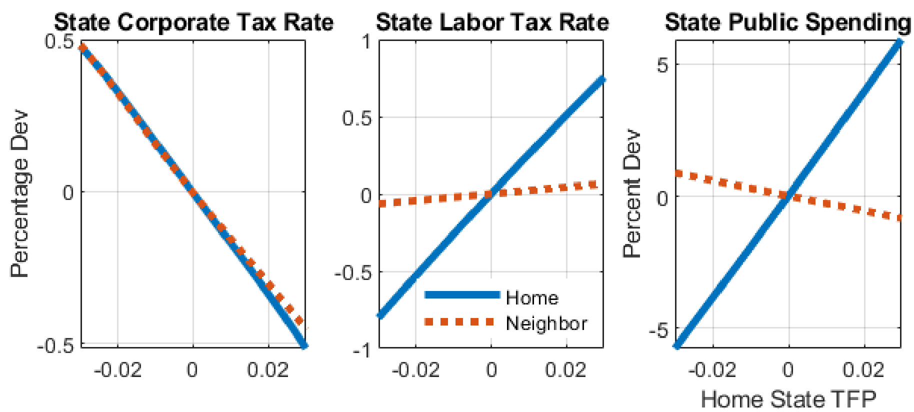

Figure 2.6: The Effect of Z_i: State Policies

In the layer of states, it can be seen from figure 2.6 that the corporate tax rates in the productive state and the normal state decline as z_i goes up. Part of the driving force is that the federal tax rates move in the direction that increases τ_k. Moreover, the magnitudes of the responses are almost the same in the two states. In the productive state, the rising TFP strengthens the allocation effect (the MPK becomes higher) and dominates the price effect. In the normal state, the outflow of capital enhances both the allocation effect and price effect. The middle graph of the above figure shows the functions of the state labor tax rates. In the productive state (state i), the labor tax rate increases since the inflow of households increase the marginal benefit of raising $\tau_{l,i,t}$ (there are more taxpayers and cannot move immediately), and the federal corporate (labor) tax rate increases (decreases). In the normal state (state j), the labor income tax rate also increases moderately because of the federal policy and the outflow of production factors that tightens the budget constraint. In the right-most graph, the state public spending represents the states' budget situation. Not surprisingly, the productive state's budget is more and more relaxed while the normal state's budget becomes tighter.

In the first graph of figure 2.7, rising TFP in the home state redistributes production factors (labor and capital) towards the productive state i. Such a redistribution increases the output in the home state and decreases the output in the neighboring state. The second graph that depicts the policy functions for k_i and k_j indicates that there is an inter-regional flow of capital. As can be seen from the slopes of the two functions, the increase in z_i not only attracts capital from the neighboring state but also the foreign country.

Interestingly, rising z_i increases both states' household welfare. It is not surprising that the home state's residents can enjoy higher welfare. But the

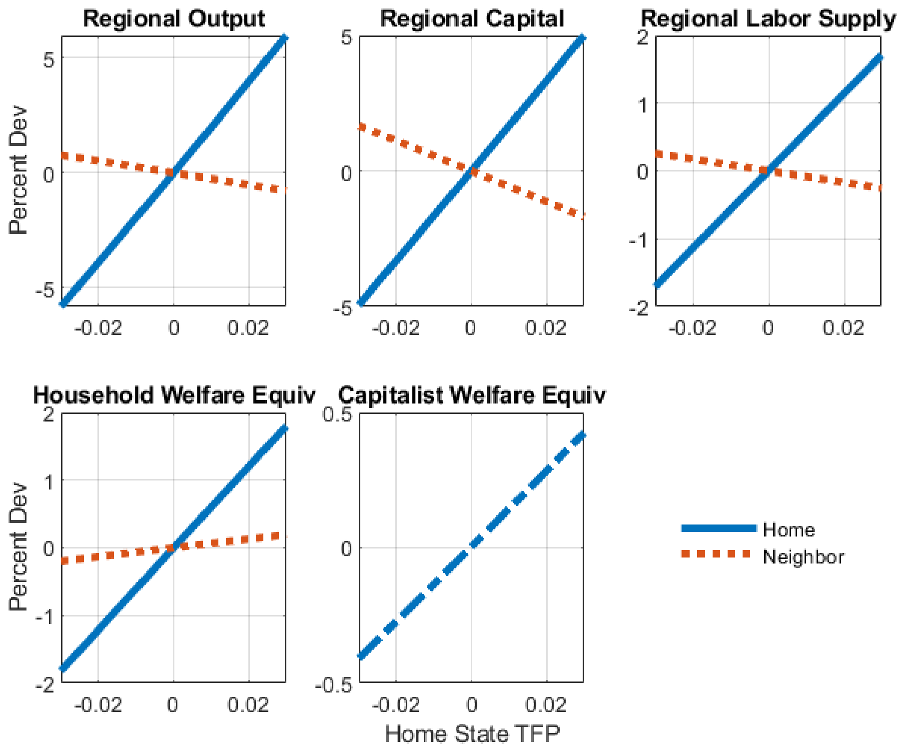

Figure 2.7: The Effect of Z_i: Economic Outcome

neighboring state that experiences factor outflow also ends up with slightly higher welfare. Indeed, the period utility provided by the neighboring state — $U(c,h) + I(\hat{g})$ — does decline, however, two other components of $V_{H,j}$ increases. First, since the shock is persistent, a fraction of households living in state j can move to state i in the next period and enjoy high welfare. Second and probably most importantly, the federal government provides a higher amount of non-rivalrous and non-excludable public goods. $V_{H,j}$ becomes higher because $\tilde{I}(G)$ and $\beta E_t p_{j,t+1} V_{H,i,t+1}$ go up with z_i. Lastly, the capitalist's welfare is also an increasing function of z_i due to the higher rate of return and dividend.

2.5 Conclusion

This paper examines the optimal fiscal policies in a federation. To model the intergovernmental fiscal relation, I introduce a Stackelberg-like framework with policy commitment in which the federal government is the leader who internalizes the states' choices and the state governments are the followers who take the federal policies as given. Capital can flow across regions costlessly. Households can migrate to achieve higher welfare with some random cost.

In this paper, I find that when capital depreciation is deductible, in the interior-point steady-state the overall tax rate on corporate income (capital) is a decreasing function of the federal corporate tax rate under widely accepted values of depreciation rate. The famous zero capital taxation result still holds in the interior point steady-state under an IES bigger than 1. However, it does not mean that each layer of government imposes zero tax rate. Instead, the federal government imposes a positive and high tax rate while the states impose negative rates. If the federal government must impose the same tax rate on labor income, the zero tax result holds when the two states have permanent TFP differences. If state-specific tax rates are available, the productive state imposes a higher corporate tax rate than the unproductive state does, but the tax-base-weighted average overall tax rate is still zero in the steady-state.

Besides, this paper delivers some interesting welfare implications regarding labor mobility and the inter-state TFP gap. Specifically, if the social welfare function is utilitarian, the total social welfare function is an increasing function of the inter-state TFP gap. This is because migration gives households (workers) the possibility to move to the productive state. Moreover, although it can improve the unproductive state's welfare, allowing non-linear federal labor income tax might not be desirable from a utilitarian point of view. Because it lowers the welfare of living in the productive state, reduces the number of households that could have migrated, and the cost is higher than the benefit it brings.

In the end, the quantitative result of my paper supports the Biden Tax Plan. It shows that increasing the federal corporate income tax rate is the optimal response to the increase in foreign country's TFP. Moreover, it finds that an increase in the regional TFP increases households' welfare in both states. The reason is that a fraction of the current residents in the unproductive state have the chance to migrate and live in the productive state in the next period, and the federal government provides a higher amount of federal public goods.

In the future, other fiscal policies such as public infrastructure, debt issuance, could be considered. They are going to rich the current analysis.

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Appendix A

Supporting Materials for Chapter 1

A.1 Empirical Analysis on Horizontal Competition

In the benchmark specification, the dependent variable is the average tax rate computed by dividing tax revenue to the tax base. The key independent variable is the weighted average tax rate such that $\tau_{-i,t} = \sum_{j\neq i} w_{i,j}\tau_{j,t}$. In this paper, the spatial weight $w_{i,j}$ is the average of inverse physical and economic distances. The intuition is that a state is more likely to compete with states whose economic development level is similar or whose population center is nearby. The economic distance is defined as the log difference of real GDP per capita, and the physical distance is the distance between state population centroids. The control variables include population, political preference (from Chirinco and Wilson, 2017), lagged government debt, and GDP per capita for both the home state and the neighboring state. To clean the simultaneity, I use the neighbor's lagged government debt, population, and political preference as instruments. Nominal variables are deflated by GDP deflator (2012=100).

Table A.1 shows the result in other specifications.

Table A.1: Result of Alternative Specifications

Variables	Corp.	Pers.	Corp.	Pers.	Pers.
Corp. Tax Rate$_{-i,t}$			0.5371	1.0562	
			(0.2663)**	(0.2104)***	
Corp. Tax Rate$_{-i,t-1}$	0.2972				
	(0.1154)*				
Pers. Tax Rate$_{-i,t}$			0.4211	-1.8050	-1.6291
			(0.3565)	(0.2817)***	(0.3016)***
Pers. Tax Rate$_{-i,t-1}$		-0.5046			
		(0.1179)***			
IV	No	No	Yes	Yes	Yes
Control	Yes	Yes	Yes	Yes	Yes
Fixed Effect	Yes	Yes	Yes	Yes	Yes
Time Effect	Yes	Yes	Yes	Yes	Yes
$N \times T$	924	924	924	924	924

In column (1) and (2), the independent variable is the neighbor's tax rate in the previous year because it might take some time for a state to react. In column (3) and (4), both corporate and personal tax rates of the neighboring state are included. In column (5), the variable of interest is the marginal individual income tax rate. I use TAXSIM(V32) to compute the marginal tax rate of a hypothetical taxpayer who files income tax jointly, has no dependent, and has household income at the median level of a state. It is difficult to find the marginal corporate tax rate, the closest variable is the one computed by Chirinko and Wilson (2017), see column A of their table 2 for the coefficient.

A.2 Equilibrium Conditions of Capital Market

For the sake of convenience, I omit the time subscription in this part. The capital market equilibrium is characterized by the following equations ($\iota \equiv o_k + \tau_k$)

$$(1-\iota_i)\left(F_{k,i}-\delta\right) = (1-\iota_{-i})\left(F_{k,-i}-\delta\right),$$

$$k_i + k_{-i} = K,$$

By using the implicit function theorem, one could have

$$\frac{\partial k_i}{\partial \tau_{k,i}} = \frac{F_{k,i}-\delta}{(1-\iota_i)F_{kk,i}+(1-\iota_{-i})F_{kk,-i}} < 0;$$

$$\frac{\partial k_i}{\partial \tau_{k,-i}} = \frac{-\left(F_{k,-i}-\delta\right)}{(1-\iota_i)F_{kk,i}+(1-\iota_{-i})F_{kk,-i}} > 0;$$

$$\frac{\partial k_i}{\partial l_i} = \frac{-\left(1-\iota_i\right)F_{kl,i}}{(1-\iota_i)F_{kk,i}+(1-\iota_{-i})F_{kk,-i}} > 0;$$

$$\frac{\partial k_i}{\partial l_{-i}} = \frac{\left(1-\iota_{-i}\right)F_{kl,-i}}{(1-\iota_i)F_{kk,i}+(1-\iota_{-i})F_{kk,-i}} < 0;$$

$$\frac{\partial k_i}{\partial K} = \frac{\left(1-\iota_{-i}\right)F_{kk,-i}}{(1-\iota_i)F_{kk,i}+(1-\iota_{-i})F_{kk,-i}} > 0.$$

Therefore, the capital used in state i can be written as:

$$k_i = K\left(\iota_i, \iota_{-i}, l_i, l_{-i}, z_i, z_{-i}, K\right).$$

Plug it into the capital demand function, the equilibrium capital rate of return equals:

$$r = R\left(\iota_i, \iota_{-i}, l_i, l_{-i}, z_i, z_{-i}, K\right).$$

A.3 Derivations of the First-Order Conditions

The objective function of a state government can be written as

$$\max\ U\left(c_{i,t},h_{i,t}\right)+I\left(g_{i,t}n_{i,t}^{-\eta}\right)+\beta E_t\left[\left(1-\Phi_{i,t+1}(\bar{\kappa}_{i,t+1})\right)V_{i,t+1}+\int_{-\infty}^{\bar{\kappa}_{i,t+1}}\left(V_{-i,t+1}-\kappa\right)d\Phi_{i,t+1}(\kappa)\right]$$

subject to:

$\psi_{i,t}$:

$$-\frac{U_{h,i,t}}{U_{c,i,t}}h_{i,t}+(1-\beta)\left(1+r_t\right)\frac{A_t}{\bar{n}}-c_{i,t}-\bar{\tau}=0;$$

$\mu_{i,t}$:

$$(1-f_t)z_{i,t}k_{i,t}^{\alpha}l_{i,t}^{1-\alpha}-\delta(1-o_{k,t})k_{i,t}+b_{i,t+1}+n_{i,t}\bar{\tau}-g_{i,t}-(1+r_t)b_{i,t}+n_{i,t}\frac{U_{h,i,t}}{U_{c,i,t}}h_{i,t}-r_tk_{i,t}-\Gamma(b_{i,t+1})=0.$$

$\theta_{i,t}$:

$$\beta(1+r_t)A_t-A_{t+1}=0.$$

The solution is characterized by the following conditions.

FONC for $g_{i,t}$:

$$I'_{i,t}n_{i,t}^{-\eta}=\mu_{i,t}.$$

FONC for $c_{i,t}$:

$$U_{c,i,t}+\left(\mu_{i,t}n_{i,t}-\psi_{i,t}\right)h_{i,t}\frac{U_{ch,i,t}U_{c,i,t}-U_{cc,i,t}U_{h,i,t}}{U_{c,i,t}^2}=\psi_{i,t}.$$

Solve the FONC for $\tau_{k,i,t}$,

$$\zeta_{k,i,t} = \frac{\partial r_t/\partial \tau_{k,i,t}}{\partial k_{i,t}/\partial \tau_{k,i,t}} \left(k_{i,t} + b_{i,t} - \frac{\beta\theta_{i,t} + (1-\beta)\psi_{i,t}/\bar{n}}{\mu_{i,t}} A_t \right),$$

in which:

$$\zeta_{k,i,t} \equiv \left[\tau_{k,i,t} + (1-\alpha)\left(o_{k,t} - o_{p,t}\right)\right] F_{k,i,t} - \tau_{k,i,t}\delta.$$

$\zeta_{k,i,t}$ measures the marginal effect of $k_{i,t}$ on the budget of government i. Divide both sides by $(F_{k,i,t} - \delta)$ and move $(1-\alpha)(o_{k,t} - o_{p,t})F_{k,i,t}/(F_{k,i,t} - \delta)$ to the right-hand-side, one could get equation (12).

Solve the FONC for $h_{i,t}$,

$$U_{h,i,t} + (n_{i,t}\mu_{i,t} - \psi_{i,t}) \left(\frac{U_{h,i,t}}{U_{c,i,t}} + \frac{U_{hh,i,t}U_{c,i,t} - U_{ch,i,t}U_{h,i,t}}{U_{c,i,t}^2} h_{i,t} \right)$$

$$+\mu_{i,t} n_{i,t} \left(1 - f_t\right) F_{l,i,t} = 0.$$

When $U(c,h) = U\left(c - \frac{h^{1+1/\varphi}}{1+1/\varphi}\right)$, the FONC for $c_{i,t}$ can be reduced to $U_{c,i,t} = \psi_{i,t}$ and $U_{h,i,t} = -\psi_{i,t} h_{i,t}^{1/\varphi}$ because the income effect of labor supply is eliminated. Divide this equation by $n_{i,t}\mu_{i,t}$ and use the fact that $h_{i,t}^{1/\varphi} = \left(1 - o_{p,t} - \tau_{p,i,t}\right) F_{l,i,t}$, one could get equation (13).

FONC for $b_{i,t}$:

$$\mu_{i,t}\left[1 - \Gamma'(b_{i,t+1})\right] - \theta_{i,t} + \beta E_t \left[\left(1 - \Phi_{i,t+1}\left(\bar{\kappa}_{i,t+1}\right)\right) \frac{\partial V_{i,t+1}}{\partial b_{i,t+1}} + \Phi_{i,t+1}\left(\bar{\kappa}_{i,t+1}\right) \frac{\partial V_{-i,t+1}}{\partial b_{i,t+1}} \right] = 0.$$

FONC for K_{t+1}:

$$-\theta_{i,t} + \beta E_t \left[(1 - \Phi_{i,t+1}(\bar{\kappa}_{i,t+1})) \frac{\partial V_{i,t+1}}{\partial K_{t+1}} + \Phi_{i,t+1}(\bar{\kappa}_{i,t+1}) \frac{\partial V_{-i,t+1}}{\partial K_{t+1}} \right] = 0.$$

A.4 Algorithm for the Quantitative Solution

To standardize the problem, I re-define $\mathbf{X}_t$ as

$$\mathbf{X}_t \equiv [n_{i,t-1}, z_{i,t}, z_{-i,t}, b_{i,t-1}, b_{-i,t-1}, o_{k,t}, o_{p,t}, K_t],$$

which is a 1-by-8 vector. I describe the computational algorithm below.

I. Use Smolyak approximation to pick the co-location points, call the set of points $\mathcal{X}$.

II. For each $\mathbf{X}_t \in \mathcal{X}$, guess the vector $\omega_t \equiv [v_{i,t}, v_{-i,t}, \tau_{k,i,t}, \tau_{k,-i,t}, h_{i,t}, h_{-i,t}, b_{i,t}, b_{-i,t}]$.[1] Call the resulting matrix Ω_0.

III. Compute the coefficients of the Chebyshev polynomials on the basis of $\mathcal{X}$ and Ω_0.

IV. For each $\mathbf{X}_t \in \mathcal{X}$, use the coefficients to solve the updated ω_t. Call the resulting matrix Ω_1. The detailed steps are as follows:

   - Given $\mathbf{X}_t$ and ω_t, compute $[n_{i,t},\ n_{-i,t},\ K_{t+1}]$.
   - Generate nodes and weights for vector $[\varepsilon_{z,i,t+1},\ \varepsilon_{z,-i,t+1},\ \varepsilon_{k,t+1},\ \varepsilon_{p,t+1}]$.
   - For each node s:

[1]For the sake of simplicity, let $a_t \equiv a(\mathbf{X}_t)$.

- * Use the law of motion for z, o_k, and o_p to compute $z_{i,t+1}$, $z_{-i,t+1}$, $o_{k,t+1}$, and $o_{p,t+1}$.
- * Construct $\mathbf{X}_{t+1}$ and compute ω_{t+1} according to the coefficients.
- * Compute $\left[V_{b_i,i,t+1}, V_{b_i,-i,t+1}, V_{b_{-i},i,t+1}, V_{b_{-i},-i,t+1}, V_{K,i,t+1}, V_{K,-i,t+1}\right]$.

– Compute the right-hand-side of the necessary conditions for government debt and K using $E_t a_{t+1} \simeq \sum_s w_s a_{s,t+1}$. Pin down $\mu_{i,t}$, $\mu_{-i,t}$, $\theta_{i,t}$, and $\theta_{-i,t}$.

– Given the multipliers, solve the updated ω_t that satisfies the definition of V and the FONCs.

V. If Ω_0 is sufficiently close to Ω_1, stop the algorithm. Otherwise, update Ω_0 by

$$\Omega_0^{\text{new}} = \rho \Omega_0^{\text{old}} + (1-\rho)\Omega_1.$$

And go back to III.

To compute the term $\int_{-\infty}^{\bar{\kappa}_{i,t+1}} \kappa \phi_{i,t+1}(\kappa) d\kappa$, I use the Gauss-Chebychev quadrature to approximate it by $\sum_s \omega_s \kappa_s \phi_{i,t+1}(\kappa_s)$.

A.5 The Immobile Factor Model

This model has the same settings as before except capital and labor are immobile. Therefore, it looks as if there is only one state. Omit the state-index for simplicity, the choice variables of each state are:

$$\hat{\pi} = \left[c_t, h_t, k_{t+1}, g_t, b_{t+1}, r_t\right]_{t=0}^{\infty}.$$

Since $o_{k,t}$ and z_t are exogenous, and K_t is predetermined, the Ramsey planner can effectively choose r_t through manipulating $\tau_{k,t}$ and $h_{i,t}$.

The optimization problem can be written as

$$\max_{\hat{\pi}} \; E_0 \sum_{t=0}^{\infty} \beta^t \left[U\left(c_t - \frac{h_t^{1+1/\varphi}}{1+1/\varphi} \right) + I\left(g_t\right) \right]$$

subject to:

$$\psi_t : \; h_t^{1+1/\varphi} + (1-\beta)(1+r_t)(k_t+b_t) - \bar{\tau} - c_t = 0,$$

$$\mu_t : \; (1-f_t) z_t k_t^{\alpha} h_t^{1-\alpha} - \delta(1-o_{k,t})k_t + b_{t+1} - g_t - (1+r_t)b_t - h_t^{1+1/\varphi} - r_t k_t - \Gamma(b_{t+1}) = 0.$$

The necessary condition for r_t reads as:

$$1 - \frac{\beta\theta_t + (1-\beta)\psi_t}{\mu_t} = 0.$$

In the immobile-factor model, the state planner chooses the tax rate to balance the price effect.

The necessary condition for b_{t+1} reads as:

$$\mu_t \left[1 - \Gamma'(b_{t+1})\right] - \theta_t = -\beta E_t \frac{\partial V_{t+1}}{\partial b_{t+1}} = \beta E_t \left[\mu_{t+1} - \beta\theta_{t+1} - (1-\beta)\psi_{t+1}\right](1 + r_{t+1}).$$

The necessary condition for A_{t+1} reads as:

$$\theta_t = \beta E_t \frac{\partial V_{t+1}}{\partial A_{t+1}} = \beta E_t \{ \left[\beta\theta_{t+1} + (1-\beta)\psi_{t+1}\right](1 + r_{t+1}) + \mu_{t+1}\zeta_{k,t+1} \}.$$

Appendix B

Supporting Materials for Chapter 2

B.1 Capital Market Equilibrium

The capital market equilibrium is characterized by the following equations (for simplicity I omit the time subscriptions):

$$k_i + k_j + k_{\mathrm{w}} = K,$$

$$r_t = (1 - T_{k,s})\,(z_s F_{1,s} - \delta)\,,\ \ s = i, j, \mathrm{w}.$$

Using the implicit function theorem, one could obtain:

$$\frac{\partial k_i}{\partial T_{k,i}} = \frac{(\Omega_j + \Omega_{\mathrm{w}})(z_i F_{1,i} - \delta)}{\sum_{s \neq s'} \Omega_s \Omega_{s'}},$$

$$\frac{\partial k_i}{\partial h_i} = \frac{-(\Omega_j + \Omega_{\mathrm{w}})(1 - T_{k,i}) z_i F_{12,i} n_i}{\sum_{s \neq s'} \Omega_s \Omega_{s'}},$$

$$\frac{\partial k_i}{\partial K} = \frac{\Omega_j \Omega_{\mathrm{w}}}{\sum_{s \neq s'} \Omega_s \Omega_{s'}}.$$

$$\frac{\partial k_i}{\partial n_i} = \frac{(\Omega_j + \Omega_{\text{w}})\Omega_i \frac{k_i}{n_i} + \Omega_j \Omega_{\text{w}} \frac{k_j}{n_j}}{\sum_{s \neq s'} \Omega_s \Omega_{s'}}$$

In which $\Omega_s \equiv (1 - T_{k,s}) z_s F_{11,s} < 0$ is the after-tax marginal product of capital in region s. In addition, the corresponding effects of T_k, h, and K on r is listed below:

$$\frac{\partial r}{\partial T_{k,i}} = -\frac{(z_i F_{1,i} - \delta)\Omega_j \Omega_{\text{w}}}{\sum_{s \neq s'} \Omega_s \Omega_{s'}} < 0,$$

$$\frac{\partial r}{\partial h_i} = \frac{(1 - T_{k,i}) F_{12,i} n_i \Omega_j \Omega_{\text{w}}}{\sum_{s \neq s'} \Omega_s \Omega_{s'}} > 0,$$

$$\frac{\partial r}{\partial K} = \frac{\Omega_i \Omega_j \Omega_{\text{w}}}{\sum_{s \neq s'} \Omega_s \Omega_{s'}} < 0,$$

$$\frac{\partial r}{\partial n_i} = \frac{\Omega_i \Omega_j \Omega_{\text{w}} \left(\frac{k_j}{n_j} - \frac{k_i}{n_i} \right)}{\sum_{s \neq s'} \Omega_s \Omega_{s'}} \ ? \ 0.$$

B.2 Derive the Federal Corporate Income Tax Rate

Starting from the necessary condition:

$$\Lambda_t \left(f_t \frac{\partial Y_{d,t}}{\partial o_{k,t}} - \delta o_{k,t} \frac{\partial K_{d,t}}{\partial o_{k,t}} \right) = \sum_s (\alpha y_{s,t} - \delta k_{s,t}) \left(n_{s,t} \mu_{s,t} - \Lambda_t \right),$$

Dividing Λ_t on both sides,

$$f_t \frac{\partial Y_{d,t}}{\partial o_{k,t}} - \delta o_{k,t} \frac{\partial K_{d,t}}{\partial o_{k,t}} = \sum_s (\alpha y_{s,t} - \delta k_{s,t}) \left(\frac{n_{s,t} \mu_{s,t}}{\Lambda_t} - 1 \right)$$

$$(1 - \alpha) o_{l,t} \frac{\partial Y_{d,t}}{\partial o_{k,t}} + \alpha o_{k,t} \frac{\partial Y_{d,t}}{\partial o_{k,t}} - \delta o_{k,t} \frac{\partial K_{d,t}}{\partial o_{k,t}} = \sum_s (\alpha y_{s,t} - \delta k_{s,t}) \frac{n_{s,t} \mu_{s,t}}{\Lambda_t} - (\alpha Y_{d,t} - \delta o_{k,t} K_{d,t}).$$

Dividing $\alpha Y_{d,t} - \delta o_{k,t} K_{d,t}$ on both sides,

$$
\begin{aligned}
o_{k,t}\varepsilon_{k,t}^{o_k} + \frac{(1-\alpha)o_{l,t}\frac{\partial Y_{d,t}}{\partial o_{k,t}}}{\alpha Y_{d,t} - \delta o_{k,t}} &= \sum_s x_{y,s,t}\frac{n_{s,t}\mu_{s,t}}{\Lambda_t} - 1 \\
\frac{1}{2}o_{k,t}\varepsilon_{k,t}^{o_k} + \frac{1}{2}\frac{(1-\alpha)o_{l,t}\frac{\partial Y_{d,t}}{\partial o_{k,t}}}{\alpha Y_{d,t} - \delta o_{k,t}} &= \frac{1}{2}\sum_s x_{y,s,t}\frac{n_{s,t}\mu_{s,t}}{\Lambda_t} - \frac{1}{2} \\
&= \text{COV}\left(x_{y,s,t}, \frac{n_{s,t}\mu_{s,t}}{\Lambda_t}\right) + Ex_{y,s,t}E\frac{n_{s,t}\mu_{s,t}}{\Lambda_t} - \frac{1}{2} \\
&= \text{COV}\left(x_{y,s,t}, \frac{n_{s,t}\mu_{s,t}}{\Lambda_t}\right) + \frac{1}{2}E\left(\frac{n_{s,t}\mu_{s,t}}{\Lambda_t} - 1\right)
\end{aligned}
$$

$$
o_{k,t} = \frac{\text{COV}\left(x_{y,s,t}, \frac{n_{s,t}\mu_{s,t}}{\Lambda_t}\right) + \frac{1}{2}E\left(\frac{n_{s,t}\mu_{s,t}}{\Lambda_t} - 1\right) - \frac{1}{2}\frac{(1-\alpha)o_{l,t}\frac{\partial Y_{d,t}}{\partial o_{k,t}}}{\alpha Y_{d,t} - \delta o_{k,t}}}{\frac{1}{2}\varepsilon_{k,t}^{o_k}}
$$

In which $\varepsilon_{k,t}^{o_k} \equiv \frac{\partial \alpha Y_{d,t} - \delta K_{d,t}}{\partial o_{k,t}} \frac{1}{\alpha Y_{d,t} - \delta K_{d,t}}$ is the semi-elasticity of the tax base with respect to $o_{k,t}$. It measures the percent change of the tax base in response to an 1 percentage increase in o_k. Using similar steps, one could obtain the corresponding result of $o_{l,t}$.

One could use similar steps to derive the federal labor income tax rate.

B.3 Derive the State-Specific Federal Labor Tax Rate

Note that the new federal budget constraint reads as:

$$
f_{i,t}y_{i,t} + f_{j,t}y_{j,t} - \delta o_{k,t}K_{d,t} = G_t.
$$

Where $f_{s,t} = (1-\alpha)o_{l,s,t} + \alpha o_{k,t}$. Taking derivative with respect to $o_{l,s,t}$,

$$
\Lambda_t \left[(1-\alpha)y_{s,t} + f_{s,t}\frac{\partial y_{s,t}}{\partial o_{l,s,t}} + f_{s',t}\frac{\partial y_{s',t}}{\partial o_{l,s,t}} - \delta o_{k,t}\frac{\partial K_{d,t}}{\partial o_{l,s,t}}\right] = n_{s,t}\mu_{s,t}(1-\alpha)y_{s,t}.
$$

Divide Λ_t on both sides,

$$f_{s,t}\frac{\partial y_{s,t}}{\partial o_{l,s,t}} + f_{s',t}\frac{\partial y_{s',t}}{\partial o_{l,s,t}} - \delta o_{k,t}\frac{\partial K_{d,t}}{\partial o_{l,s,t}} = \left(\frac{n_{s,t}\mu_{s,t}}{\Lambda_t} - 1\right)(1-\alpha)y_{s,t},$$

Divide $(1-\alpha)y_{s,t}$ and use $f_{s,t} = \alpha o_{k,t} + (1-\alpha)o_{l,s,t}$,

$$o_{l,s,t}\varepsilon_{l,s,t}^{o_l} + \frac{\alpha o_{k,t}\frac{\partial y_{s,t}}{\partial o_{l,s,t}} + f_{s',t}\frac{\partial y_{s',t}}{\partial o_{l,s,t}} - \delta o_{k,t}\frac{\partial K_{d,t}}{\partial o_{l,s,t}}}{(1-\alpha)y_{s,t}} = \frac{n_{s,t}\mu_{s,t}}{\Lambda_t} - 1$$

$$\Rightarrow o_{l,s,t} = \frac{\left(\frac{n_{s,t}\mu_{s,t}}{\Lambda_t} - 1\right) - \frac{\alpha o_{k,t}\frac{\partial y_{s,t}}{\partial o_{l,s,t}} + f_{s',t}\frac{\partial y_{s',t}}{\partial o_{l,s,t}} - \delta o_{k,t}\frac{\partial K_{d,t}}{\partial o_{l,s,t}}}{(1-\alpha)y_{s,t}}}{\varepsilon_{l,s,t}^{o_l}}.$$

B.4 Description of the Solution Algorithm

This part describes the algorithm that I use to find the solution. The big picture is to use Projection method to approximate the corresponding functions. Since $\mathbf{X}_t$ is a 7-by-1 vector, I use Smolyak method to reduce the number of grid points to be taken care of and ease the curse of dimensionality (denote $\mathcal{X}$ the set of co-location grid points). The detailed algorithm is described as follows. For every $\mathbf{X}_t \in \mathcal{X}$, make initial guesses of $V_{H,s}^{(0)}(\mathbf{X}_t)$, $\hat{V}_C^{(0)}(\mathbf{X}_t)$ and compute $\mathbf{n}(\mathbf{X}_t)$. Given this, make a initial guess of $\mathbf{o}(\mathbf{n}_t, \mathbf{X}_t)$. Compute the coefficients of the polynomials. In iteration k, run the following steps:

- For each point $\mathbf{X}_t \in \mathcal{X}$
  - given the polynomial coefficients obtained from iteration $k-1$, solve the state-level policy functions and the corresponding $\hat{V}_{H,s,t}$ and $\hat{V}_{C,t}$

for given value of $\hat{x}_t$:

$$\hat{x}_t \equiv [n_{1,t}, z_{1,t}, z_{2,t}, z_{w,t}, o_{k,t}, o_{l,t}, K_t, \theta_{i,t}, \theta_{j,t}]$$

and compute the coefficients of the polynomials;

- for $s = i, j$, compute $\mathbf{n}_t$ according to equation (1) and $V_{H,s}^{(k-1)}(\mathbf{X}_t)$;
- given the results in step (1.1), pick $\mathbf{o}^*$ that maximizes[1] the social welfare and compute the corresponding $V_{H,s,t}$ ($s = i, j$) and $V_{C,t}$.

- Stack the result and call them $M^k(\mathcal{X}) \equiv \left[V_{H,i}^{(k)}(\mathcal{X}), V_{H,j}^{(k)}(\mathcal{X}), V_C^{(k)}(\mathcal{X}), O^k(\mathcal{X})\right]$.
- Stop the algorithm if $M^{k-1}(\mathcal{X})$ and $M^k(\mathcal{X})$ are sufficiently close.[2] Otherwise, return to step one.

Here are the details of step (1.1). First, select the co-location points and make a initial guess of $\pi_{i,t}$, $\pi_{j,t}$, D_t, $\theta_{i,t}$, and $\theta_{j,t}$ and compute the corresponding coefficients of the polynomials. In iteration number k, run the following steps:

- For each co-location point $\hat{x}_t$:

  (a) Use the $\pi_{s,t}^{(k)}$ ($s = i, j$) to compute $A_{d,t+1}$, $\theta_{i,t+1}$, and $\theta_{j,t+1}$.

  (b) For each node (denoted by m) of Gauss-Quadrature, construct x_{t+1}^m, use the policy function computed in the previous iteration to obtain $\mathbf{n}_{t+1}^m$, $\mathbf{o}_{t+1}^m$, and use the resulting coefficients to compute $\pi_{s,t+1}^m$ ($s = i, j$).

[1] I use the built-in function "fmincon" to find the combination that minimizes $-W$.
[2] For the value functions, the tolerance is 10^{-6}. For the federal tax rate, the tolerance is 10^{-4}.

(c) Compute the right-hand-side of the forward-looking conditions according to the definitions and compute the summations — they are the approximations of expectations.

(d) Given the right-hand-side of the forward-looking conditions, compute the new $\pi_{s,t}^{(k+1)}$ $(s = i, j)$ according to the first necessary conditions.

- Stop the algorithm if the updated policy is sufficiently close to the previous iteration (the tolerance is 10^{-6}), otherwise return to the step first step.